OPE analysis of the nucleon scattering tensor including weak interaction and finite mass effects

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Abstract

We perform a systematic operator product expansion of the most general form of the nucleon scattering tensor $W_{\mu\nu}$ including electromagnetic and weak interaction processes. Finite quark masses are taken into account and a number of higher-twist corrections are included. In this way we derive relations between the lowest moments of all 14 structure functions and matrix elements of local operators. Besides reproducing well-known results, new sum rules for parity-violating polarized structure functions and new mass correction terms are obtained.

PACS numbers: 12.38.-t; 11.50.Li; 13.85.Hd UFTP preprint 404/1995

I. INTRODUCTION

Deep inelastic lepton-nucleon scattering (DIS) provides the cleanest method to understand the internal structure of the nucleon [1]. Presently such measurements performed at HERA, CERN, SLAC, and other accelerators concentrate on electromagnetic DIS. Theoretically weak interaction processes can provide valuable additional information [2], allowing e.g. a flavor decomposition of structure functions. Such experiments are extremely difficult and require careful and detailed analysis, but the possible scientific progress justifies a serious consideration of them [3].

All DIS experiments aim at measuring the structure functions of the nucleon. In the framework of operator product expansion (OPE), moments of structure functions can be related to nucleon matrix elements of well defined local operators which have to be calculated nonperturbatively, e.g. in lattice QCD [4]. In some cases those matrix elements occur in other strong interaction processes and sum rules can be derived. The polarized Bjorken sum rule [5], relating nucleonic β-decay and the first moment of the polarized spin structure function $\int dx g_1(x)$, is an especially interesting case. Such sum rules offer a convincing way of testing perturbative and non perturbative QCD.

With the accuracy of QCD tests reaching the per cent or even per mille level, the higher-twist contributions to DIS become relevant. While quite a number of theoretical investigations on their form exists scattered in the literature, no systematic presentation of all contributions up to the second order in the expansion of the fermion propagator within the formalism of OPE has been given so far. As compared to the interactionless quark parton model used in the analysis of [3], the OPE formalism is of general validity and includes the QCD interactions. This can be seen in the fact that within the framework of the quark parton model [3] obtains for example $g_2^\gamma(x) = 0$, while the OPE predicts $\int_0^1 dx x^2 g_2^\gamma(x)$ being different from zero.

This paper gives a complete determination of the lowest moments of the 14 nucleon structure functions which parametrize the most general form of the nucleon scattering tensor (only restricted by Lorentz and time-reversal invariance). The paper is organized as follows: In section II we define all relevant quantities and follow the standard procedure of OPE to isolate contributions of well defined twist. This section is supplemented by appendix A in which the decomposition of all relevant operators is performed explicitly. Using these results we analyze γ and Z^0 exchange in section III and $W^\pm$ exchange in section IV. Our final results are presented in section V and the conclusions in section VI. A discussion of the contributions from cat-ear diagrams is delegated to appendix B.

Our main results are new higher-twist and quark mass corrections to the parity violating polarized structure functions.

II. DEFINITIONS AND SPIN DECOMPOSITION

DIS processes can be distinguished according to the exchanged boson, namely γ, Z^0, or $W^\pm$. The cross sections for these processes are

$$\frac{d^2\sigma^{(\gamma)}}{dE'd\Omega} = \frac{E'}{EM}\frac{\alpha^2}{(Q^2)^2}L^{(\gamma)}_{\mu\nu}W^{(\gamma)\mu\nu} \quad , \tag{1}$$

$$\frac{d^2\sigma^{(Z^0)}}{dE'd\Omega} = \frac{E'}{EM}\frac{\alpha^2}{(Q^2+(M_{Z^0})^2)^2}L^{(Z^0)}_{\tau\sigma}W^{(Z^0)}_{\mu\nu}\left(-g^{\mu\tau}+\frac{q^\mu q^\tau}{M^2_{Z^0}}\right)\left(-g^{\nu\sigma}+\frac{q^\nu q^\sigma}{M^2_{Z^0}}\right) \quad , \tag{2}$$

$$\frac{d^2\sigma^{(W^\pm)}}{dE'd\Omega} = \frac{E'}{EM}\frac{\alpha^2}{(Q^2+(M_{W^\pm})^2)^2}L^{(W^\pm)}_{\tau\sigma}W^{(W^\pm)}_{\mu\nu}\left(-g^{\mu\tau}+\frac{q^\mu q^\tau}{M^2_{W^\pm}}\right)\left(-g^{\nu\sigma}+\frac{q^\nu q^\sigma}{M^2_{W^\pm}}\right) \quad . \tag{3}$$

Here E is the energy of the incoming lepton, E' the energy of the outgoing lepton, and M is the nucleon mass. Ω is the solid angle of the outgoing lepton. As usual q is the photon, respectively boson momentum with $Q^2 = -q^2$, α is the electromagnetic

coupling constant. The weak interaction coupling constants are absorbed into the lepton ($L_{\mu\nu}$) and nucleon ($W_{\mu\nu}$) scattering tensors.

These tensors can be written in the following form:

$$
\begin{aligned}
L^{(Pr)}_{\mu\nu} &= \sum_{\sigma'} \langle k\sigma| \left(j^{(L,Pr)}_\mu(0)\right)^\dagger |k'\sigma'\rangle\langle k'\sigma'|j^{(L,Pr)}_\nu(0)|k\sigma\rangle \quad , \\
\left(W^{(Pr)}_{\mu\nu}\right)_\lambda &= \frac{1}{2\pi}\sum_X (2\pi)^4\delta^4(p_X-p-q)\frac{1}{2}\langle p\lambda| \left(j^{(H,Pr)}_\mu(0)\right)^\dagger |X\rangle\langle X|j^{(H,Pr)}_\nu(0)|p\lambda\rangle \\
&= \frac{1}{4\pi}\int d^4\xi e^{iq\xi}\langle p\lambda| \left[\left(j^{(H,Pr)}_\mu(\xi)\right)^\dagger, j^{(H,Pr)}_\nu(0)\right]_- |p\lambda\rangle \quad .
\end{aligned}
\tag{4}
$$

The index $Pr = \gamma, Z^0, W^+, W^-$ indicates the process under consideration. The lepton current is labeled by L and the hadronic one by H. k^μ is the four-momentum of the incoming lepton and σ^μ its spin. k'^μ and σ'^μ are four-momentum and spin of the outgoing lepton. The nucleon four-momentum is p^μ, ($p^2 = M^2$) and $\lambda = \pm 1/2$ denotes its polarization (cf. figure 1). Let S_μ be its spin vector , then according to [10]

$$
W^{(Pr)}_{\mu\nu}(q,p,S) = \sum_\lambda \langle\lambda|\hat\rho|\lambda\rangle \left(W^{(Pr)}_{\mu\nu}(q,p)\right)_\lambda \quad , \tag{5}
$$

with the spin density matrix

$$
\hat\rho := \frac{1}{2}\left(1+\vec\sigma\cdot\frac{\vec S}{M}\right) \quad , \tag{6}
$$

and $S^\mu = (0,\vec S)$ in the rest frame of the target ($S^2 = -M^2$, $S\cdot p = 0$). We use covariant normalization for the lepton states $\langle k\sigma|k'\sigma'\rangle = 2k_0(2\pi)^3\delta(\vec k - \vec k')\delta_{\sigma,\sigma'}$, and $\langle pS|p'S'\rangle = 2p_0(2\pi)^3\delta(\vec p - \vec p')\delta_{S,S'}$ for the nucleon states in $W^{(PR)}_{\mu\nu}(q,p,S)$. The lepton currents read

$$
\begin{aligned}
j^{(L,\gamma)}_\mu(\xi) &= \sum_e \overline{e}(\xi)\gamma_\mu e(\xi) \quad , \\
j^{(L,Z^0)}_\mu(\xi) &= \sum_l \overline{l}(\xi)\gamma_\mu(V'_l + A'_l\gamma_5)l(\xi) \quad , \\
j^{(L,W^+)}_\mu(\xi) &= \sum_{e,\nu} \overline{e}(\xi)\gamma_\mu(V'^+ + A'^+\gamma_5)\nu(\xi) \quad , \\
j^{(L,W^-)}_\mu(\xi) &= \sum_{e,\nu} \overline{\nu}(\xi)\gamma_\mu(V'^- + A'^-\gamma_5)e(\xi) \quad ,
\end{aligned}
\tag{7}
$$

where $l \in \{e^-, \mu^-, \tau^-, \nu_e, \nu_\mu, \nu_\tau\}$, $e \in \{e^-, \mu^-, \tau^-\}$ and $\nu \in \{\nu_e, \nu_\mu, \nu_\tau\}$. The nucleon current is written as

$$
\begin{aligned}
j_\mu^{(H,\gamma)}(\xi) &= \sum_f Q_f \overline{q}_f(\xi)\gamma_\mu q_f(\xi) \quad , \\
j_\mu^{(H,Z^0)}(\xi) &= \sum_f \overline{q}_f(\xi)\gamma_\mu(V_f + A_f\gamma_5)q_f(\xi) \quad , \\
j_\mu^{(H,W^+)}(\xi) &= \overline{q}_u(\xi)\gamma_\mu(V_{ud}^+ + A_{ud}^+\gamma_5)q_d(\xi) + \text{further flavor combinations} \quad , \\
j_\mu^{(H,W^-)}(\xi) &= \overline{q}_d(\xi)\gamma_\mu(V_{du}^- + A_{du}^-\gamma_5)q_u(\xi) + \text{further flavor combinations} \quad .
\end{aligned} \tag{8}
$$

The constants V and A follow from the standard model (see for example [11]). Q_f is the electromagnetic quark charge and f runs over all flavors. All quark states considered here are mass eigenstates, i.e. Cabbibo-Kobayashi-Maskawa mixing [12] is suppressed for the sake of a compact notation. To simplify notation we suppress the obvious flavor indices $V_{ud}^\pm \to V^\pm$ in the following.

A slight modification arises for the γ and Z^0 interference terms.

$$
\begin{aligned}
\frac{d^2\sigma^{(\text{int. }\gamma,Z^0)}}{dE'd\Omega} &= \frac{E'}{EM}\frac{\alpha^2}{(Q^2+(M_{Z^0})^2)Q^2}\left(-g^{\mu\tau}\right)\left(-g^{\nu\sigma}+\frac{q^\nu q^\sigma}{M_{Z^0}^2}\right) \\
&\quad \times \left(L_{\tau\sigma}^{(\text{int. }\gamma,Z^0)(1)}W_{\mu\nu}^{(\text{int. }\gamma,Z^0)(1)} + L_{\sigma\tau}^{(\text{int. }\gamma,Z^0)(2)}W_{\nu\mu}^{(\text{int. }\gamma,Z^0)(2)}\right) \quad , \\
\left(W_{\mu\nu}^{(\text{int. }\gamma,Z^0)(1)}\right)_\lambda &= \frac{1}{4\pi}\int d^4\xi e^{iq\xi}\langle p\lambda|\left[\left(j_\mu^{(H,\gamma)}(\xi)\right)^\dagger, j_\nu^{(H,Z^0)}(0)\right]_-|p\lambda\rangle \quad , \\
\left(W_{\mu\nu}^{(\text{int. }\gamma,Z^0)(2)}\right)_\lambda &= \frac{1}{4\pi}\int d^4\xi e^{iq\xi}\langle p\lambda|\left[\left(j_\mu^{(H,Z^0)}(\xi)\right)^\dagger, j_\nu^{(H,\gamma)}(0)\right]_-|p\lambda\rangle \quad , \\
L_{\mu\nu}^{(\text{int. }\gamma,Z^0)(1)} &= \sum_{\sigma'}\langle k\sigma|\left(j_\mu^{(L,\gamma)}(0)\right)^\dagger|k'\sigma'\rangle\langle k'\sigma'|j_\nu^{(L,Z^0)}(0)|k\sigma\rangle \quad , \\
L_{\mu\nu}^{(\text{int. }\gamma,Z^0)(2)} &= \sum_{\sigma'}\langle k\sigma|\left(j_\mu^{(L,Z^0)}(0)\right)^\dagger|k'\sigma'\rangle\langle k'\sigma'|j_\nu^{(L,\gamma)}(0)|k\sigma\rangle \quad , \\
\left(W_{\mu\nu}^{(\text{int. }\gamma,Z^0)}\right)_\lambda &= \left(W_{\mu\nu}^{(\text{int. }\gamma,Z^0)(1)}\right)_\lambda + \left(W_{\mu\nu}^{(\text{int. }\gamma,Z^0)(2)}\right)_\lambda \quad , \\
L_{\mu\nu}^{(\text{int. }\gamma,Z^0)} &= L_{\mu\nu}^{(\text{int. }\gamma,Z^0)(1)} + L_{\mu\nu}^{(\text{int. }\gamma,Z^0)(2)} \quad .
\end{aligned} \tag{9}
$$

Using Lorentz covariance and time-reversal invariance the nucleon scattering tensor $W_{\mu\nu}^{(Pr)}(q, P, S)$ can be expressed in terms of 14 structure functions [2] (In our sign

convention all ϵ structures are multiplied by $-i$.):

$$
\begin{aligned}
W^{\mu\nu(Pr)}(q,p,S) = & \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)F_1^{(Pr)}(x,Q^2)+\hat{p}^\mu\hat{p}^\nu\frac{1}{\nu}F_2^{(Pr)}(x,Q^2) \\
& +q^\mu q^\nu\frac{1}{\nu}F_4^{(Pr)}(x,Q^2)+(p^\mu q^\nu+p^\nu q^\mu)\frac{1}{2\nu}F_5^{(Pr)}(x,Q^2) \\
& -i\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha}{\nu}\left[S_\beta g_1^{(Pr)}(x,Q^2)+\left(S_\beta-p_\beta\frac{Sq}{\nu}\right)g_2^{(Pr)}(x,Q^2)\right] \\
& -i\epsilon^{\mu\nu\alpha\beta}\left[\frac{p_\alpha S_\beta}{\nu}g_3^{(Pr)}(x,Q^2)+\frac{q_\alpha p_\beta}{2\nu}F_3^{(Pr)}(x,Q^2)\right] \\
& +\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\frac{Sq}{\nu}a_1^{(Pr)}(x,Q^2)+\hat{p}^\mu\hat{p}^\nu\frac{Sq}{\nu^2}a_2^{(Pr)}(x,Q^2) \\
& +q^\mu q^\nu\frac{Sq}{\nu^2}a_4^{(Pr)}(x,Q^2)+(p^\mu q^\nu+p^\nu q^\mu)\frac{Sq}{2\nu^2}a_5^{(Pr)}(x,Q^2) \\
& +\left(\hat{S}^\mu\hat{p}^\nu+\hat{S}^\nu\hat{p}^\mu\right)\frac{1}{2\nu}b_1^{(Pr)}(x,Q^2)+\left(\hat{S}^\mu p^\nu+\hat{S}^\nu p^\mu\right)\frac{1}{2\nu}b_2^{(Pr)}(x,Q^2).
\end{aligned}
\tag{10}
$$

Here $\nu = p\cdot q$ and $x = Q^2/(2\nu)$. Furthermore the vectors $\hat{p}_\mu = p_\mu + q_\mu\,\nu/Q^2$, and $\hat{S}_\mu = S_\mu + q_\mu\,S\cdot q/Q^2$, which vanish if contracted with q_μ, are introduced. Note that terms proportional to q^μ or q^ν give contributions of order m_l/E when contracted with the lepton tensor $L_{\mu\nu}$. The decomposition yields seven structure functions for parity-conserving processes and additional seven for parity-non-conserving processes. All 14 are given in table I. Six structure functions $(F_4, F_5, g_3, a_4, a_5, b_2)$ are related to non-conserved currents and are non-zero only from non-vanishing quark masses.

In the framework of OPE the moments of structure functions can be related to nucleon matrix elements of local operators with well defined twist. To this end the nucleon scattering tensor $W_{\mu\nu}^{(Pr)}$ is related to the virtual forward Compton scattering amplitude $T_{\mu\nu}^{(Pr)}$

$$
W_{\mu\nu}^{(Pr)} = \frac{1}{4\pi i}\left(T_{\mu\nu}^{(Pr)}(q_0+i\epsilon)-T_{\mu\nu}^{(Pr)}(q_0-i\epsilon)\right) \quad , \tag{11}
$$

$$
T_{\mu\nu}^{(Pr)}(q_0\pm i\epsilon) = \frac{(2\pi)^3\delta^3(\vec{p}_X-\vec{p}-\vec{q}\,)}{p_{X0}-p_0-(q_0\pm i\epsilon)}\sum_X\langle pS|\left(j_\mu^{(Pr)}\right)^\dagger(0)|X\rangle\langle X|j_\nu^{(Pr)}(0)|pS\rangle \quad , \tag{12}
$$

$$
\int_0^1 dx x^n W_{\mu\nu}^{(Pr)}(x) = \frac{1}{8\pi i}\oint\frac{T_{\mu\nu}^{(Pr)}(\omega)d\omega}{\omega^{n+2}} \quad , \tag{13}
$$

with $\omega = 1/x = 2\nu/Q^2$. In the following we deal with crossing-even and crossing-odd amplitudes

$$
\begin{aligned}
T^{(\gamma)}_{\mu\nu}(-q) &= T^{(\gamma)}_{\nu\mu}(q) \quad , \\
T^{(Z^0)}_{\mu\nu}(-q) &= T^{(Z^0)}_{\nu\mu}(q) \quad , \\
T^{(\overline{\nu}+\nu)}_{\mu\nu}(-q) &= T^{(\overline{\nu}+\nu)}_{\nu\mu}(q) \quad , \\
T^{(\overline{\nu}-\nu)}_{\mu\nu}(-q) &= -T^{(\overline{\nu}-\nu)}_{\nu\mu}(q) \quad ,
\end{aligned}
\tag{14}
$$

where $T^{(\overline{\nu}\pm\nu)}_{\mu\nu} = T^{(W^-)}_{\mu\nu} \pm T^{(W^+)}_{\mu\nu}$. Note that the formal equation (13) is only valid for certain moments of the structure functions because its derivation requires for closing the integration contour according to fig. 2 for the n-th moment

$$
T^{(Pr)}_{\mu\nu}(\omega) = -(-)^n \, T^{(Pr)}_{\mu\nu}(-\omega) \quad . \tag{15}
$$

This condition if fulfilled in the case of the crossing-even amplitudes such as $T^{(\gamma)}_{\mu\nu}$, $T^{(Z_0)}_{\mu\nu}$ and $T^{(\overline{\nu}+\nu)}_{\mu\nu}$ for the odd moments (even n) of the structure functions F_2, F_4, a_2, a_4, b_1, b_2, g_1, g_2 and F_3 and for the even moments of F_1, F_5, a_1, a_5, and g_3. For the crossing-odd amplitude $T^{(\overline{\nu}-\nu)}_{\mu\nu}$ the situation is just inverted. Furthermore performing the integration in equation (13) is possible only if $T_{\mu\nu}(\omega)$ vanishes fast enough for $\omega \to \infty$, i.e, when the integration over the circle in infinity in figure 2 does not contribute. This is true for $n \geq 1$ but not necessarily for $n = 0$. (In practice one expects nonsinglet sum rules to converge better since there is no pomeron contribution and nonsinglet structure functions are less singular for small x.)

To analyze the Compton forward scattering amplitude

$$
T^{(Pr)}_{\mu\nu}(q,p,S) = i \int d^4\xi e^{iq\xi} \langle pS| T\left((j^{(Pr)}_\mu)^\dagger(\xi) j^{(Pr)}_\nu(0) \right) |pS\rangle \tag{16}
$$

we insert the nucleon currents Eq.(8) which we now write in the general form

$$j_{\mu f f'}(\xi) = \overline{q}_f(\xi)\Gamma_{\mu f f'} q_{f'}(\xi) \quad . \tag{17}$$

Note that $\Gamma_{\mu f f'} = \Gamma_{\mu f' f}$. Restricting ourselves for the moment to the handbag diagram (cf. figure 3) we write

$$\begin{aligned} T_{\mu\nu f f'} = i \int d^4\xi e^{iq\xi} \langle pS| \Big\{ & \overline{q}_f(\xi)\Gamma_{\mu f f'} i S_{f'}(\xi,0)\Gamma_{\nu f f'} q_f(0) \\ & + \overline{q}_{f'}(0)\Gamma_{\nu f f'} i S_f(0,\xi)\Gamma_{\mu f f'} q_{f'}(\xi) \Big\} |pS\rangle \quad . \end{aligned} \tag{18}$$

Here $S_f(\xi,0)$ is the fermionic propagator in the external field approximation for a quark with flavor f. In the Schwinger formalism for calculations in external fields as proposed by Shuryak and Vainshtein [6], (18) can be written as

$$\begin{aligned} T_{\mu\nu f f'} = -\langle pS| \Bigg\{ & \overline{q}_f(0)\Gamma_{\mu f f'} \frac{1}{\not{P} - m_{f'} + \not{q}} \Gamma_{\nu f f'} q_f(0) \\ & + \overline{q}_{f'}(0)\Gamma_{\nu f f'} \frac{1}{\not{P} - m_f - \not{q}} \Gamma_{\mu f f'} q_{f'}(0) \Bigg\} |pS\rangle \quad , \end{aligned} \tag{19}$$

with $P_\mu = i\partial_\mu + gA_\mu$. Such an expression is very conveniently expanded in powers of $1/Q$

$$\frac{1}{\not{P} - m_f - \not{q}} = -\frac{1}{\not{q}} \sum_{j=0}^{\infty} \left((\not{P} - m_f) \frac{1}{\not{q}} \right)^j \quad . \tag{20}$$

This is the starting point of our OPE analysis. Inserting the expansion in (19) it immediately becomes clear that for $f = f'$ all odd terms in j are symmetric under exchange of μ and ν and thus contribute to the unpolarized structure functions, while all even terms in j are antisymmetric contributing to the polarized structure functions.

Finally, to define twist the different Lorentz structures are classified according to their spin. Although this is standard in principle some ambiguities enter for more than two Lorentz indices. To avoid confusion we state explicitly the decomposition we use in appendix A.

III. PHOTON AND Z^0 EXCHANGE

In this section we perform the decomposition of the virtual forward Compton scattering amplitude for Z^0 exchange. The corresponding expression for γ exchange can simply be obtained by setting $V_f \to Q_f$ and $A_f \to 0$. For parity-even quantities γ and Z^0 exchange always interfere. The interference term is given by

$$T_{\mu\nu}^{(\mathrm{int}\ \gamma,\mathrm{Z}^0)} = \left(T_{\mu\nu}^{(\gamma+Z^0)}\right)_{\substack{V_f \to V_f + Q_f \\ A_f \to A_f}} - \left(T_{\mu\nu}^{(\gamma)}\right)_{\substack{V_f \to Q_f \\ A_f \to 0}} - \left(T_{\mu\nu}^{(Z^0)}\right)_{\substack{V_f \to V_f \\ A_f \to A_f}} \quad . \tag{21}$$

We will now proceed as in [6] and [15] except for using a different classification of matrix elements. The virtual forward Compton scattering amplitude for γ and Z^0 exchange reads

$$\begin{aligned} T_{\mu\nu}^{(\gamma,Z^0)} = -\sum_f \langle pS| \Big\{ & \overline{q}_f(0)\gamma_\mu[V_f + A_f\gamma_5]\frac{1}{\not{P} - m_f + \not{q}}\gamma_\nu[V_f + A_f\gamma_5]q_f(0) \\ + \quad & \overline{q}_f(0)\gamma_\nu[V_f + A_f\gamma_5]\frac{1}{\not{P} - m_f - \not{q}}\gamma_\mu[V_f + A_f\gamma_5]q_f(\xi)\Big\}|pS\rangle \quad . \end{aligned} \tag{22}$$

Expanding in $(\not{P} - m_f)/Q$ according to (20) we get for the zeroth order term

$$\begin{aligned} T_{\mu\nu}^{(\gamma,Z^0)}\Big|_{0.\ \text{order}} &= -\sum_f \langle pS| \Big\{ \overline{q}_f(0)\gamma_\mu[V_f + A_f\gamma_5]\frac{\not{q}}{q^2}\gamma_\nu[V_f + A_f\gamma_5]q_f(0) \\ &\qquad - \overline{q}_f(0)\gamma_\nu[V_f + A_f\gamma_5]\frac{\not{q}}{q^2}\gamma_\mu[V_f + A_f\gamma_5]q_f(0)\Big\}|pS\rangle \\ &= -2i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{\nu}\omega a_{f5}^{(0,2)}(V_f^2 + A_f^2) \\ &\quad -2i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{2\nu}\omega[2a_f^{(0,2)}](2V_fA_f) \quad , \end{aligned} \tag{23}$$

where the identity

$$\gamma_\mu\gamma_\alpha\gamma_\nu = g_{\mu\alpha}\gamma_\nu + g_{\nu\alpha}\gamma_\mu - g_{\mu\nu}\gamma_\alpha - i\epsilon_{\mu\alpha\nu\sigma}\gamma_5\gamma^\sigma \tag{24}$$

was used and the following matrix elements were defined:

$$\begin{aligned} 2a_{f5}^{(0,2)}S^\sigma &= \langle pS|\overline{q}_f(0)\gamma^\sigma\gamma_5 q_f(0)|pS\rangle \quad , \\ 2a_f^{(0,2)}p^\sigma &= \langle pS|\overline{q}_f(0)\gamma^\sigma q_f(0)|pS\rangle \quad . \end{aligned} \tag{25}$$

The classification of matrix elements is to be understood as follows

$$a_{f5-}^{(m,t)}:\begin{cases} m = & \text{number of } P_\mu - \text{operators.} \\ t = & \text{twist.} \\ f = & \text{quark flavor.} \\ 5 = & \text{indicates } \gamma_5. \\ - = & \text{indicates odd chirality, left out} \\ & \text{in case of even chirality.} \end{cases} \tag{26}$$

The spin of the operator is $s = 3 + m - t$. The chirality of an operator is said to be even if

$$\overline{q}_f \gamma_5 \hat{O} \gamma_5 q_f = -\overline{q}_f \hat{O} q_f \tag{27}$$

holds. The other case, with the positive sign, indicates chiral odd operators. The contribution of first order is symmetric in μ and ν. It reads:

$$\begin{aligned} T_{\mu\nu}^{(\gamma,Z^0)}\Big|_{\text{1. order}} = & \sum_f \langle pS|\Big\{\overline{q}_f(0)\gamma_\mu[V_f + A_f\gamma_5]\frac{\not{q}}{q^2}(\not{P} - m_f)\frac{\not{q}}{q^2}\gamma_\nu[V_f + A_f\gamma_5]q_f(0) \\ & +\overline{q}_f(0)\gamma_\nu[V_f + A_f\gamma_5]\frac{\not{q}}{q^2}(\not{P} - m_f)\frac{\not{q}}{q^2}\gamma_\mu[V_f + A_f\gamma_5]q_f(0)\Big\}|pS\rangle \quad . \end{aligned} \tag{28}$$

We decompose the above expression into operators of well defined spin, which are parametrized in terms of reduced matrix elements. We write the result in a way matching the decomposition structure of the nucleon scattering tensor $W_{\mu\nu}$. In this way we get

$$\begin{aligned} T_{\mu\nu}^{(\gamma,Z^0)}\Big|_{\text{1. order}} = & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\left[\left(2\omega^2 a_f^{(1,2)} + 4\frac{M^2}{Q^2}a_f^{(1,2)}\right)(V_f^2 + A_f^2)\right. \\ & \left. -4\frac{m_f M}{Q^2}a_{f-}^{(0,3)}(V_f^2 - A_f^2)\right] \\ & +\frac{1}{\nu}\hat{p}_\mu\hat{p}_\nu\left[4\omega a_f^{(1,2)}(V_f^2 + A_f^2)\right] + \frac{1}{\nu}q_\mu q_\nu\left[4\omega\frac{m_f M}{Q^2}a_{f-}^{(0,3)}A_f^2\right] \\ & +\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\frac{Sq}{\nu}\left[2\omega^2 a_{f5}^{(1,2)}\right]2V_f A_f + q_\mu q_\nu\frac{Sq}{\nu^2}\left[-\omega^3\frac{m_f}{M}a_{f5-}^{(0,2)}\right]2V_f A_f \end{aligned}$$

$$+\frac{Sq}{2\nu^2}(p_\mu q_\nu + p_\nu q_\mu)\left[-2\omega^2\frac{m_f}{M}a^{(0,2)}_{f5-}\right]2V_fA_f$$
$$+\frac{1}{2\nu}(\hat{S}_\mu\hat{p}_\nu + \hat{S}_\nu\hat{p}_\mu)\left[4\omega a^{(1,2)}_{f5} + 4\omega\frac{m_f}{M}a^{(0,2)}_{f5-}\right]2V_fA_f$$
$$+\frac{1}{2\nu}(\hat{S}_\mu p_\nu + \hat{S}_\nu p_\mu)\left[-4\omega\frac{m_f}{M}a^{(0,2)}_{f5-}\right]2V_fA_f \quad . \tag{29}$$

For the matrix element $a^{(0,2)}_{f5-}$ we employed the equation of motion. The matrix elements are again classified according to Eq.(26). The exact form of the operators can be found in appendix A.

The contribution of second order in $1/Q$ is antisymmetric in μ and ν. It reads

$$T^{(\gamma,Z^0)}_{\mu\nu}\Big|_{2.\text{ order}} = -\sum_f \langle pS|\overline{q}_f(0)\gamma_\mu[V_f + A_f\gamma_5]\frac{\not{q}}{q^2}(\not{P} - m_f)\frac{\not{q}}{q^2}(\not{P} - m_f)\frac{\not{q}}{q^2}\gamma_\nu[V_f + A_f\gamma_5]q_f(0)$$
$$-\overline{q}_f(0)\gamma_\nu[V_f + A_f\gamma_5]\frac{\not{q}}{q^2}(\not{P} - m_f)\frac{\not{q}}{q^2}(\not{P} - m_f)\frac{\not{q}}{q^2}\gamma_\mu[V_f + A_f\gamma_5]q_f(0)|pS\rangle \tag{30}$$

and can be decomposed into

$$T^{(\gamma,Z^0)}_{\mu\nu}\Big|_{2.\text{ order}} = -\frac{1}{q^6}\sum_f\Big[-8i\epsilon_{\mu\nu\lambda\sigma}q_\alpha q_\beta q^\lambda\langle pS|\overline{q}_f(0)P^\alpha P^\beta\gamma^\sigma q_f(0)\gamma_5|pS\rangle(V_f^2 + A_f^2)$$
$$+4q^2q^\lambda\epsilon_{\mu\nu\lambda\sigma}\epsilon^\sigma{}_{\tau\rho\delta}\langle pS|\overline{q}_f(0)P^\tau P^\rho\gamma^\delta q_f(0)|pS\rangle(V_f^2 + A_f^2)$$
$$+2q^2m_fq^\lambda\epsilon_{\mu\nu\lambda\sigma}\epsilon^\sigma{}_{\tau\rho\delta}\langle pS|\overline{q}_f(0)\gamma^\tau\gamma^\rho P^\delta q_f(0)|pS\rangle(V_f^2 + A_f^2)$$
$$+4q^2m_fq_\alpha\langle pS|\overline{q}_f(0)P^\alpha[\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu]q_f(0)|pS\rangle A_f^2$$
$$-4i\epsilon_{\mu\nu\lambda\sigma}q^2q^\lambda m_f^2\langle pS|\overline{q}_f(0)\gamma^\sigma\gamma_5q_f(0)|pS\rangle(V_f^2 + A_f^2)$$
$$-8i\epsilon_{\mu\nu\lambda\sigma}q_\alpha q_\beta q^\lambda\langle pS|\overline{q}_f(0)P^\alpha P^\beta\gamma^\sigma q_f(0)|pS\rangle 2A_fV_f$$
$$+4q^2q^\lambda\epsilon_{\mu\nu\lambda\sigma}\epsilon^\sigma{}_{\alpha\beta\gamma}\langle pS|\overline{q}_f(0)P^\alpha P^\beta\gamma^\gamma\gamma_5|pS\rangle 2A_fV_f\Big] \quad . \tag{31}$$

Here we use the following relations that result from the equation of motion $\not{P}q_f = m_fq_f$:

$$m_f\langle pS|\overline{q}_f(0)\gamma_\mu q_f(0)|pS\rangle = \langle pS|\overline{q}_f(0)P_\mu q_f(0)|pS\rangle \quad ,$$
$$\langle pS|\overline{q}_f(0)\gamma_5q_f(0)|pS\rangle = 0 \quad ,$$

$$\langle pS|\overline{q}_f(0)P_\mu\gamma_5 q_f(0)|pS\rangle = 0 \quad ,$$
$$\langle pS|\overline{q}_f(0)(P_\mu\gamma_\nu\gamma_\lambda + P_\nu\gamma_\lambda\gamma_\mu + P_\lambda\gamma_\mu\gamma_\nu)\gamma_5 q_f(0)|pS\rangle = 0 \quad . \tag{32}$$

The totally antisymmetric spin-0 part has to be treated separately. It can be transformed in the following way (Note, that these equations are only valid when sandwiched between $\langle pS|\overline{q}_f(0)$ and $q_f(0)|pS\rangle$.):

$$\epsilon^{\sigma\alpha\beta\gamma}P_\alpha P_\beta\gamma_\gamma = i(P^2 - m_f^2)\gamma^\sigma\gamma_5 \quad , \tag{33}$$
$$\epsilon^{\sigma\alpha\beta\gamma}P_\alpha P_\beta\gamma_\gamma\gamma_5 = i(P^2 - m_f^2)\gamma^\sigma \quad , \tag{34}$$
$$\epsilon^{\sigma\alpha\beta\gamma}\gamma_\alpha\gamma_\beta P_\gamma = 2im_f\gamma^\sigma\gamma_5 \quad , \tag{35}$$
$$\epsilon^{\sigma\alpha\beta\gamma}\gamma_\alpha\gamma_\beta P_\gamma\gamma_5 = 0 \quad . \tag{36}$$

With the definition of the dual gluonic field strength tensor $ig\tilde{G}_{\sigma\tau} = i\frac{g}{2}\epsilon_{\sigma\tau\alpha\beta}G^{\alpha\beta} = \frac{1}{2}\epsilon_{\sigma\tau\alpha\beta}[P_\alpha, P_\beta]$ and the equation of motion the operators $P^2\gamma^\sigma$ and $P^2\gamma^\sigma\gamma_5$ can be transformed into a mass component and a gluonic component. In terms of matrix elements this transformation reads:

$$m_f^2 a_{f5}^{(0,2)} + M^2 a_{f5}^{(2,4)} = M^2\tilde{a}_{f5}^{(2,4)} \quad , \tag{37}$$
$$m_f^2 a_f^{(0,2)} + M^2 a_f^{(2,4)} = M^2\tilde{a}_f^{(2,4)} \quad , \tag{38}$$

where the reduced matrix elements are listed in appendix A. Here also a second relation is derived:

$$a_{f5}^{(2,3)} = \tilde{a}_{f5}^{(2,3)} - \frac{m_f}{M}a_{f5-}^{(1,2)} \quad . \tag{39}$$

Collecting all terms we get for the second order of our expansion in $(\not{P} - m_f)/Q$:

$$T_{\mu\nu}^{(\gamma,Z^0)}\Big|_{2.\ order} = -i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{\nu}\left[\frac{2}{3}\omega^3 a_{f5}^{(2,2)} + \frac{4}{3}\omega^3\tilde{a}_{f5}^{(2,3)} + \frac{4}{9}\frac{M^2}{Q^2}\omega\left(a_{f5}^{(2,2)} + 4\tilde{a}_{f5}^{(2,3)} + 4\tilde{a}_{f5}^{(2,4)}\right)\right.$$
$$\left. -\frac{8}{3}\frac{1}{Q^2}\omega a_{f5}^{(0,2)}m_f^2\right](V_f^2 + A_f^2)$$

$$-i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{\nu}\left[-\frac{4}{3}\frac{m_f^2}{Q^2}\omega a_{f5}^{(0,2)}\right](V_f^2 - A_f^2)$$

$$-i\epsilon_{\mu\nu\lambda\sigma}Sq\frac{q^\lambda p^\sigma}{\nu^2}\left[\frac{4}{3}\omega^3(a_{f5}^{(2,2)} - \tilde{a}_{f5}^{(2,3)})(V_f^2 + A_f^2)\right]$$

$$-i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{2\nu}\left[4\omega^3 a_f^{(2,2)} + \frac{8}{9}\frac{M^2}{Q^2}\omega\left(3a_f^{(2,2)} + 8\tilde{a}_f^{(2,4)}\right)\right.$$
$$\left.-\frac{88}{9}\frac{1}{Q^2}\omega a_f^{(0,2)} m_f^2\right] 2V_f A_f \quad . \tag{40}$$

IV. $W^{\pm}$-EXCHANGE

In the case of W^- exchange we use the current

$$j_\mu^{(W^-)}(\xi) = \overline{d}(\xi)\gamma_\mu(V - A\gamma_5)u(\xi) + \dots \quad , \tag{41}$$

where the ellipsis denotes currents involving heavier quarks. For simplicity we will discuss only the $u \to d$ current. For other flavors the relations look just the same. The equation analogous to (22) is then given by

$$\begin{aligned} T_{\mu\nu}^{(W^-)} &= i\int d^4\xi e^{iq\xi}\langle pS|T\Big(\overline{u}(\xi)\gamma_\mu(V + A\gamma_5)d(\xi)\overline{d}(0)\gamma_\nu(V + A\gamma_5)u(0)\Big)|pS\rangle \\ &= -\langle pS|\Big\{\overline{u}(0)\gamma_\mu(V + A\gamma_5)\frac{1}{\not{P} - m_d + \not{q}}\gamma_\nu(V + A\gamma_5)u(0) \\ &\qquad +\overline{d}(0)\gamma_\nu(V + A\gamma_5)\frac{1}{\not{P} - m_u - \not{q}}\gamma_\mu(V + A\gamma_5)d(0)\Big\}|pS\rangle \quad , \end{aligned} \tag{42}$$

where the expansion in $\not{P} - m_f$ becomes more complicated since two different flavors are involved. In each order symmetric and antisymmetric terms appear. For the zeroth order term of the expansion we have

$$\begin{aligned} T_{\mu\nu}^{(W^-)}\Big|_{0.\ \mathrm{order}} &= -\langle pS|\{\overline{u}(0)\gamma_\mu(V + A\gamma_5)\frac{\not{q}}{q^2}\gamma_\nu(V + A\gamma_5)u(0) \\ &\qquad +\overline{d}(0)\gamma_\nu(V + A\gamma_5)\frac{\not{q}}{q^2}\gamma_\mu(V + A\gamma_5)d(0)\}|pS\rangle \end{aligned}$$

$$
\begin{aligned}
&= \Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \omega a_u^{(0,2)} + \frac{1}{2\nu}(p_\mu q_\nu + p_\nu q_\mu)[2\omega a_u^{(0,2)}] + \frac{1}{\nu} q_\mu q_\nu \frac{1}{2}\omega^2 a_u^{(0,2)} \\
&\quad - i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda S^\sigma}{\nu} \omega a_{u5}^{(0,2)} \Bigg\} (V^2 + A^2) \\
&\quad + \Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{Sq}{\nu} \omega a_{u5}^{(0,2)} + \frac{1}{2\nu}(\hat{S}_\mu \hat{p}_\nu + \hat{S}_\nu \hat{p}_\mu)[4a_{u5}^{(0,2)}] \\
&\quad - \frac{1}{2\nu}(\hat{S}_\mu p_\nu + \hat{S}_\nu p_\mu)[4a_{u5}^{(0,2)}] + \frac{Sq}{\nu^2} q_\mu q_\nu \left[-\frac{1}{2}\omega^2 a_{u5}^{(0,2)} \right] \\
&\quad - i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda p^\sigma}{2\nu} \omega [2a_u^{(0,2)}] \Bigg\} 2VA + (u \leftrightarrow d, \mu \leftrightarrow \nu, q \leftrightarrow -q) \quad .
\end{aligned} \tag{43}
$$

The fact that in first order of the expansion we get contributions to current non-conserved structure functions, namely F_4, F_5, b_2, and a_4 is due to an incomplete definition of $T_{\mu\nu}$. Indeed, in zeroth order no quark masses appear and consequently, both the vector and the axial vector current should be conserved, requiring $q^\mu T_{\mu\nu}|_{0.\ order} = 0$. The terms proportional to F_4, F_5, b_2, and a_4 are therefore unphysical and must be subtracted. In this sense our result (43) has to be rewritten in a form where only current-conserved contributions occur:

$$
\begin{aligned}
T_{\mu\nu}^{(W^-)}\Big|_{0.\ \text{order}} &= \Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \omega a_u^{(0,2)} \quad - i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda S^\sigma}{\nu} \omega a_{u5}^{(0,2)} \Bigg\} (V^2 + A^2) \\
&\quad + \Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \frac{Sq}{\nu} \omega a_{u5}^{(0,2)} + \frac{1}{2\nu}(\hat{S}_\mu \hat{p}_\nu + \hat{S}_\nu \hat{p}_\mu)[4a_{u5}^{(0,2)}] \\
&\quad - i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda p^\sigma}{2\nu} \omega [2a_u^{(0,2)}] \Bigg\} 2VA + (u \leftrightarrow d, \mu \leftrightarrow \nu, q \leftrightarrow -q) \quad .
\end{aligned} \tag{44}
$$

This phenomenon arises only in the first order of expansion. Formally it comes from unphysical contact terms (seagull terms) generated by derivatives acting on the theta function in the time ordered product [16]. In this way we obtain, expanding the propagator to first order

$$
\begin{aligned}
T_{\mu\nu}^{(W^-)}\Big|_{1.\ \text{order}} &= \langle pS| \{ \overline{u}(0)\gamma_\mu (V + A\gamma_5) \frac{\not{q}}{q^2} (\not{P} - m_d) \frac{\not{q}}{q^2} \gamma_\nu (V + A\gamma_5) u(0) \\
&\quad + \overline{d}(0)\gamma_\nu (V + A\gamma_5) \frac{\not{q}}{q^2} (\not{P} - m_d) \frac{\not{q}}{q^2} \gamma_\mu (V + A\gamma_5) d(0) \} |pS\rangle
\end{aligned}
$$

$$
\begin{aligned}
&= \Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \left[\omega^2 a_u^{(1,2)} + 2\frac{M^2}{Q^2} a_u^{(1,2)}\right] \\
&\quad + \frac{1}{\nu}\hat{p}_\mu \hat{p}_\nu [2\omega a_u^{(1,2)}] + \frac{1}{\nu} q_\mu q_\nu \left[\frac{m_u M}{Q^2}\omega a_{u-}^{(0,3)}\right]\Bigg\} (V^2 + A^2) \\
&\quad + \Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \frac{Sq}{\nu}[\omega^2 a_{u5}^{(1,2)}] + \frac{Sq}{2\nu^2}(p_\mu q_\nu + p_\nu q_\mu)[-\omega^2 \frac{m_u}{M} a_{u5-}^{(0,2)}] \\
&\quad + \frac{Sq}{\nu^2} q_\mu q_\nu \left[-\frac{1}{2}\omega^3 \frac{m_u}{M} a_{u5-}^{(0,2)}\right] + \frac{1}{2\nu}(\hat{S}_\mu \hat{p}_\nu + \hat{S}_\nu \hat{p}_\mu)[2\omega(a_{u5}^{(1,2)} + \frac{m_u}{M} a_{u5-}^{(0,2)})] \\
&\quad + \frac{1}{2\nu}(\hat{S}_\mu p_\nu + \hat{S}_\nu p_\mu)[-2\omega \frac{m_u}{M} a_{u5-}^{(0,2)}]\Bigg\} 2VA \\
&\quad + \Bigg(-i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda S^\sigma}{\nu} \left[\frac{\omega^2}{2}(a_{u5}^{(1,2)} + \frac{m_u}{M} a_{u5-}^{(0,2)})\right] \\
&\qquad -i\epsilon_{\mu\nu\lambda\sigma} Sq \frac{q^\lambda p^\sigma}{\nu^2} \left[\frac{\omega^2}{2}(a_{u5}^{(1,2)} - \frac{m_u}{M} a_{u5-}^{(0,2)})\right] \\
&\qquad -i\epsilon_{\mu\nu\lambda\sigma} \frac{p^\lambda S^\sigma}{\nu} [\omega \frac{m_u}{M} a_{u5-}^{(0,2)}]\Bigg) (V^2 + A^2) \\
&\qquad -i\epsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda p^\sigma}{2\nu} [2\omega^2 a_u^{(1,2)}] 2VA \\
&\quad + (u \leftrightarrow d, \mu \leftrightarrow \nu) \quad .
\end{aligned} \tag{45}
$$

The expression for the second order term

$$
\begin{aligned}
T_{\mu\nu}^{(W^-)}\Big|_{\text{2. order}} &= \langle pS|\{\overline{u}(0)\gamma_\mu(V + A\gamma_5)\frac{\not{q}}{q^2}(\not{P} - m_d)\frac{\not{q}}{q^2}(\not{P} - m_d)\frac{\not{q}}{q^2}\gamma_\nu(V + A\gamma_5)u(0) \\
&\quad + \overline{d}(0)\gamma_\nu(V + A\gamma_5)\frac{\not{q}}{q^2}(\not{P} - m_u)\frac{\not{q}}{q^2}(\not{P} - m_u)\frac{\not{q}}{q^2}\gamma_\mu(V + A\gamma_5)d(0)\}|pS\rangle
\end{aligned}
$$

is again decomposed into components antisymmetric and symmetric in μ and ν. The latter one reads:

$$
\begin{aligned}
T_{\mu\nu}^{(W^-)}\Big|_{\substack{\text{2. order} \\ \text{sym. in } \mu\nu}} &= \Big\{ \left(-g_{\mu\nu} + \tfrac{q_\mu q_\nu}{q^2}\right) \left[\omega^3 a_u^{(2,2)} + \tfrac{8}{9}\tfrac{M^2}{Q^2}\omega[3a_u^{(2,2)} - \tilde{a}_u^{(2,4)}]\right. \\
&+ \left.\tfrac{1}{9}\tfrac{1}{Q^2}[11m_u^2 - 9m_d^2]\omega a_u^{(0,2)}\right] + \tfrac{1}{2\nu}(p_\mu q_\nu + p_\nu q_\mu)\left[\tfrac{2}{Q^2}[m_u^2 - m_d^2]\omega a_u^{(0,2)}\right] \\
&\quad + \tfrac{1}{\nu} q_\mu q_\nu \left[\tfrac{1}{2}\tfrac{\omega^2}{Q^2}[3m_u^2 - m_d^2] a_u^{(0,2)}\right] + \tfrac{1}{\nu}\hat{p}_\mu \hat{p}_\nu \left[2\omega^2 a_u^{(2,2)}\right] \Big\} (V^2 + A^2)
\end{aligned}
$$

$$
+ \Big\{ \left(-g_{\mu\nu} + \tfrac{q_\mu q_\nu}{q^2}\right) \tfrac{Sq}{\nu} \left[\omega^3 a_{u5}^{(2,2)} + \tfrac{8}{9}\tfrac{M^2}{Q^2}\omega[a_{u5}^{(2,2)} - 2\tilde{a}_{u5}^{(2,3)} + \tilde{a}_{u5}^{(2,4)}]\right.
$$

$$+\tfrac{4}{3}\tfrac{m_u M}{Q^2}\omega a^{(1,2)}_{u5-} - \tfrac{1}{3}\tfrac{1}{Q^2}[m_u^2+3m_d^2]\omega a^{(0,2)}_{u5}\Big]$$

$$+\tfrac{Sq}{2\nu^2}(p_\mu q_\nu + p_\nu q_\mu)\left[-\tfrac{m_u}{M}\omega^3 a^{(1,2)}_{u5-}\right] + \tfrac{Sq}{\nu^2}q_\mu q_\nu\left[-\tfrac{1}{2}\tfrac{m_u}{M}\omega^4 a^{(1,2)}_{u5-} - \tfrac{1}{2}\tfrac{1}{Q^2}[m_u^2-m_d^2]\omega^2 a^{(0,2)}_{u5}\right]$$

$$+\tfrac{Sq}{\nu^2}\hat{p}_\nu\hat{p}_\nu\left[\tfrac{2}{3}\omega^2 a^{(2,2)}_{u5} - \tfrac{8}{3}\omega^2\tilde{a}^{(2,3)}_{u5} + 2\tfrac{m_u}{M}\omega^2 a^{(1,2)}_{u5-}\right]$$
$$+\tfrac{1}{2\nu}(\hat{S}_\mu\hat{p}_\nu + \hat{S}_\nu\hat{p}_\mu)\left[\tfrac{4}{3}\omega^2 a^{(2,2)}_{u5} + \tfrac{8}{3}\omega^2\tilde{a}^{(2,3)}_{u5} + \tfrac{8}{3}\tfrac{m_u M}{Q^2}a^{(1,2)}_{u5-} + \tfrac{4}{3}\tfrac{1}{Q^2}[m_u^2-3m_d^2]a^{(0,2)}_{u5}\right]$$
$$+\tfrac{1}{2\nu}(\hat{S}_\mu p_\nu + \hat{S}_\nu p_\mu)\left[-2\tfrac{m_u}{M}\omega^2 a^{(1,2)}_{u5-} - \tfrac{8}{3}\tfrac{m_u M}{Q^2}a^{(1,2)}_{u5-} - \tfrac{4}{3}\tfrac{1}{Q^2}[m_u^2-3m_d^2]a^{(0,2)}_{u5}\right]\Big\}\,2VA$$
$$+(u\leftrightarrow d, \mu\leftrightarrow\nu, q\leftrightarrow -q) \quad . \tag{46}$$

The corresponding antisymmetric part differs from the Z^0 case only by the mass terms and, of course, by a factor 1/2.

$$T^{(W^-)}_{\mu\nu}\Big|_{\substack{\text{2. order}\\ \text{asymm. in }\mu\nu}} = \Big\{-i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda S^\sigma}{\nu}\Big[\frac{1}{3}\omega^3 a^{(2,2)}_{u5} + \frac{2}{3}\omega^3\tilde{a}^{(2,3)}_{u5}$$
$$+\frac{2}{9}\frac{M^2}{Q^2}\omega[a^{(2,2)}_{u5} + 4\tilde{a}^{(2,3)}_{u5} + 4\tilde{a}^{(2,4)}_{u5}] - \frac{1}{3}\frac{1}{Q^2}[m_u^2+3m_d^2]a^{(0,2)}_{u5}\omega\Big]$$

$$-i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{\nu^2}Sq\left[\frac{2}{3}\omega^3[a^{(2,2)}_{u5} - \tilde{a}^{(2,3)}_{u5}]\right]\Big\}(V^2+A^2)$$

$$+\Big\{-i\epsilon_{\mu\nu\lambda\sigma}\frac{q^\lambda p^\sigma}{2\nu}\Big[2\omega^3 a^{(2,2)}_u + \frac{4}{9}\frac{M^2}{Q^2}\omega[3a^{(2,2)}_u + 8\tilde{a}^{(2,4)}_u]$$
$$-\frac{1}{9}\frac{1}{Q^2}[26m_u^2 + 18m_d^2]a^{(0,2)}_u\omega\Big]\Big\}\,2VA$$

$$+(u\leftrightarrow d, \mu\leftrightarrow\nu, q\leftrightarrow -q) \quad . \tag{47}$$

We have discussed matrix elements up to twist 4. At this twist level the cat ear diagram (see figure 4) may contribute. It describes non-perturbative interactions between the hit quark and the spectator quarks in the final state. However, it turns out (see appendix B) that the matrix elements of the cat ear diagram yield contributions of structure functions that are one order in $1/Q^2$ higher such that we

do not have to include them in the order we are working. Also, due to symmetry properties cat ear diagrams can only contribute to F_1, F_2, a_1, and b_2, as discussed in appendix B.

V. SUM RULES

Having analyzed the virtual forward Compton scattering amplitude completely up to order $[(\not{P} - m_f)/Q]^2$, we have determined the lowest moments of the 14 structure functions. The results are compiled in table II and the resulting sum rules are listed in table III on the leading twist level. The mass corrections to these sum rules can be read off from table II so we do not repeat them. Table IV gives some relations between structure functions, valid in the limit $Q^2 \to \infty$, i.e. neglecting higher-twist corrections. In general, here we do not consider possible anomalous contributions arising from the divergence of pseudovector currents.

Let us add some comments to these tables:

1.) The index f runs over all quark and antiquark flavors, i.e. $f = u, \overline{u}, d, \overline{d}, \ldots$ Due to the opposite helicity of quarks and antiquarks we have

$$V_f = V_{\overline{f}} \quad ; \qquad A_f = -A_{\overline{f}} \quad . \tag{48}$$

Furthermore, we use in the case of neutrino scattering for the weak coupling constants the abbreviation

$$\eta_W = \frac{1}{8 \sin^2 \theta_W} = \frac{G}{\sqrt{2}} \frac{M_W^2}{4\pi\alpha} \quad , \tag{49}$$

and for brevity, in the case of $W^{\pm}$ - exchange, we introduce the following flavor combinations: S (singlet), V (valence) and furthermore the combinations ΔS and ΔV.

$$a_S := \eta_W[(a_u + a_{\overline{u}}) + (a_d + a_{\overline{d}}) + (\text{further generations})] \;,$$

$$a_V := \eta_W[(a_u - a_{\overline{u}}) + (a_d - a_{\overline{d}}) + (\text{further generations})] \;,$$

$$a_{\Delta S} := \eta_W[(a_u + a_{\overline{u}}) - (a_d + a_{\overline{d}}) + (\text{further generations})] \;,$$

$$a_{\Delta V} := \eta_W[(a_u - a_{\overline{u}}) - (a_d - a_{\overline{d}}) + (\text{further generations})] \;. \tag{50}$$

The relations between these flavor combinations and the symmetries of the amplitudes are presented in table V.

To simplify the mass terms we define

$$(m_S - m'_S)\, a_S = \eta_W[(m_u - m_d)\, a_u + (m_{\overline{u}} - m_{\overline{d}})\, a_{\overline{u}}$$
$$+ (m_d - m_u)\, a_d + (m_{\overline{d}} - m_{\overline{u}})\, a_{\overline{d}} + (\text{further generations})] \;, \tag{51}$$

and analogous expressions for V, Δ S, and Δ V.

The constant C appearing in table III for the polarized Bjorken sum rule has the value

$$C = \underbrace{\frac{1}{3}}_{\gamma} + \underbrace{\frac{1}{3\sin 2\theta_W}[4\sin\theta_W - 1]}_{\gamma - Z^0 \text{ interference}} + \underbrace{\frac{2}{3}\frac{\sin\theta_W}{\sin^2 2\theta_W}[2\sin\theta_W - 1]}_{Z^0} \quad . \tag{52}$$

Isospin T_3 and baryon number B are given by

$$\sum_{f=u,c,t}\left(a_f^{(0,2)} - a_{\overline{f}}^{(0,2)}\right) - \sum_{f=d,s,b}\left(a_f^{(0,2)} - a_{\overline{f}}^{(0,2)}\right) = 2T_3 \quad , \tag{53}$$

$$\sum_{f=u,c,t}\left(a_u^{(0,2)} - a_{\overline{f}}^{(0,2)}\right) + \sum_{f=d,s,b}\left(a_f^{(0,2)} - a_{\overline{f}}^{(0,2)}\right) = 3B \quad , \tag{54}$$

Finally we note that we do not include here the radiative corrections and chose for simplicity the normalization point to be $\mu^2 = Q^2$.

2.) The twist-2 matrix element of the tensor operator $(-i)\sigma_{\mu\nu}$ determines the transversity distribution h_1 [17] and does not appear at leading twist in deep inelastic lepton nucleon scattering. As a chiral-odd operator it is related to spin-flip

processes which are always suppressed by the quark mass. As may be expected beforehand this operator determines the polarized structure function g_3 which is related to a non-conserved current and vanishes identically for massless quarks.

3.) The simplest scalar twist-3 operator is the scalar operator

$$\langle pS|\overline{q}_f(0)q_f(0)|pS\rangle = 2a_{f-}^{(0,3)}M \quad , \tag{55}$$

which for the flavor combination $\frac{1}{2}(m_u+m_d)(\bar{u}u+\bar{d}d)$ is known as the σ-term, giving rise to a sum rule for F_4 which, however is expected to be divergent due to a Regge analysis [17]. Since the scalar operator is chiral odd it will appear in deep inelastic scattering only if quark masses are not ignored.

4.) The twist-3 quark-gluon-quark correlator

$$\begin{aligned} &\langle pS|\overline{q}_f(0)\frac{1}{6}\left[(\gamma_\alpha g\tilde{G}_{\beta\sigma} + \gamma_\beta g\tilde{G}_{\alpha\sigma})\right]q_f(0)|pS\rangle - \text{traces} \\ &= 2a_{f5}^{(2,3)}\left[\frac{1}{3}(2p_\alpha p_\beta S_\sigma - p_\beta p_\sigma S_\alpha - p_\sigma p_\alpha S_\beta) - \text{traces}\right] \end{aligned} \tag{56}$$

(often referred to as $d^{(2)}$) contributes to the first moment of g_1 and the third moment of g_2. This operator contributes also higher-twist terms to a_1 and a_2.

5.) The twist-4 quark-gluon-quark correlators

$$\langle pS|\overline{q}_f(0)g\tilde{G}^{\alpha\beta}\gamma_\beta q_f(0)|pS\rangle = 2a_{f5}^{(2,4)}M^2S^\alpha \quad . \tag{57}$$

contributes to g_1 and a_1 and this matrix element is commonly referred to as $f^{(2)}$. The spin independent twist-4 operator defined as

$$\langle pS|\overline{q}_f(0)g\tilde{G}^{\alpha\beta}\gamma_\beta\gamma_5 q_f(0)|pS\rangle = 2a_f^{(2,4)}M^2p^\alpha \quad , \tag{58}$$

gives a higher-twist corrections to the Gross-Llewellyn Smith sum rule. (The matrix element of the analogous twist-3 operator vanishes due to its symmetry properties,

i.e. $a_f^{(2,3)} \equiv 0$.)

6.) It is possible to relate the higher-twist contributions for the second moment of g_2 to the structure function g_3 and to obtain an interesting relation, which reads

$$\int_0^1 xdxg_2^{(\overline{\nu}-\nu)} = -\frac{1}{2}\int_0^1 xdxg_1^{(\overline{\nu}-\nu)} + \frac{1}{2}\int_0^1 dxg_3^{(\overline{\nu}-\nu)} + \mathcal{O}(1/Q^2) \quad . \tag{59}$$

7.) We cannot confirm the relation $a_2(x) = 2xa_1(x)$ for $Q^2 \to \infty$, which was claimed in [2], but rather reproduce the relation found by Ravishankar [18] and earlier by Dicus [19]:

$$2xa_1(x) = a_2(x) + b_1(x) + b_2(x) \quad . \tag{60}$$

8.) For the polarized parity-non-conserving structure functions we reproduce the relation derived by Wray [20] by means of the light-cone quark algebra:

$$\int_0^1 dx \left(a_1^{(\overline{\nu}-\nu)p} - a_1^{(\overline{\nu}-\nu)n}\right) = -2\frac{g_A}{g_V}\eta_w + \mathcal{O}(1/Q^2) \quad . \tag{61}$$

While the current non-conserved structure functions, i.e. a_4, a_5, and b_2, will hardly be measurable in practical experiments, the current conserved structure functions a_1, a_2 and b_1 should be experimentally accessible [3]. They give interesting additional information on the structure of the nucleon.

9.) We would like to stress that the operators connected with $a_{f5}^{(2,3)}$ and $a_{f5}^{(2,4)}$ measure important properties of the nucleon, namely contributions of the collective color electromagnetic field to the spin and to the momentum of the nucleon. In the rest system $S^\alpha = (0, \vec{S})$, $p^\alpha = (M, \vec{0})$ the spin-dependent operators can be written as [21]

$$\begin{aligned} \langle pS|\vec{j}_a \cdot \vec{B}|pS\rangle &= 0 \quad , \\ \langle pS|\left[-\vec{B}_a j_a^0 + (\vec{j}_a \times \vec{E}_a)\right]|pS\rangle &= 2M^2 a_{f5}^{(2,4)}\vec{S} \quad , \\ \langle pS|\left[2\vec{B}_a j_a^0 + (\vec{j}_a \times \vec{E}_a)\right]|pS\rangle &= 8M^2 a_{f5}^{(2,3)}\vec{S} \quad , \end{aligned} \tag{62}$$

where $j_a^\mu = -g\overline{q}\gamma^\mu t_a q$ denotes the quark current, B_a^σ the color magnetic field, and E_a^σ the color electric field. $a_{f5}^{(2,3)}$ and $a_{f5}^{(2,4)}$ were calculated in relativistic quark models such as the MIT bag model [22] as well as in the framework of QCD sum rules [24,23,21] making it possible to estimate the strength of the color electric and magnetic fields within the nucleon separately.

Analogously, for the axial vector current $j_{a5}^\mu = -g\overline{q}\gamma^\mu\gamma_5 t_a q$ we get

$$\begin{aligned}\langle pS|\vec{j}_{a5}\cdot\vec{B}|pS\rangle &= -2M^3 a_f^{(2,4)} \quad ,\\ \langle pS|\left[-\vec{B}_a j_{a5}^0 + (\vec{j}_{a5}\times\vec{E}_a)\right]|pS\rangle &= 0 \quad ,\\ \langle pS|\left[2\vec{B}_a j_{a5}^0 + (\vec{j}_{a5}\times\vec{E}_a)\right]|pS\rangle &= 0 \quad .\end{aligned} \tag{63}$$

The two operators $\vec{j}_{a5}\times\vec{E}_a$ and $\vec{B}_a j_{a5}^0$ determine a vector direction. As in the nucleon rest frame the only distinct direction is given by the spin, which is an axial vector, the matrix elements of $\vec{j}_{a5}\times\vec{E}_a$ and $\vec{B}_a j_{a5}^0$ must vanish. We would like to point out, that the only nonvanishing matrix element in this context, namely $a_f^{(2,4)}$, determines the higher-twist corrections to the Gross-Llewellyn Smith sum rule and was calculated in the QCD sum rule approach in [25].

VI. SUMMARY AND CONCLUSIONS

In this paper we have presented a complete analysis for the first moments of the structure functions of the nucleon scattering tensor for γ, Z^0, and $W^\pm$ exchange. All our calculations include the effects of finite quark masses.

Besides reproducing well-known results we found the relation

$$b_2(x) = 2xa_5(x) = 4x^2 a_4(x) \quad . \tag{64}$$

The extension of our work to the third order in $\not{P} - m_f$ will require a consistent treatment of the cat ear diagrams.

Acknowledgements

We thank Lech Mankiewicz for many useful discussions and continuous encouragement. This work has been supported by DFG (G. Hess Programm), Cusanuswerk, and BMBF. A.S. thanks the MPI für Kernphysik in Heidelberg for support.

APPENDIX A: DEFINITION OF THE MATRIX ELEMENTS

For the sake of brevity we introduce the notation of the totally symmetric tensor $s_{\alpha_1\alpha_2\alpha_3\alpha_4} := |\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}|$, i.e. $s_{\alpha_1\alpha_2\alpha_3\alpha_4}$ vanishes for the same index combinations as $\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}$, and whenever there is an index combination such that $\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4} = \pm 1$, then $s_{\alpha_1\alpha_2\alpha_3\alpha_4} = 1$. All other conventions are taken from [14].

i) $g^{\mu\nu}$ has spin 0.

ii) Each Lorentz vector P^μ and γ^μ has spin 1.

iii) The operator $P^\mu\gamma^\nu$ can be decomposed into a symmetric and traceless spin-2 part, an antisymmetric spin-1 part, and a spin-0 part according to

$$\begin{aligned} P^\mu\gamma^\nu &= [P^\mu\gamma^\nu]_{\text{Spin}-2} + [P^\mu\gamma^\nu]_{\text{Spin}-1} + [P^\mu\gamma^\nu]_{\text{Spin}-0} \\ &= \left[\frac{1}{2}\left(P^\mu\gamma^\nu + P^\nu\gamma^\mu\right) - \frac{1}{4}g^{\mu\nu}\not{P}\right] + \left[\frac{1}{2}\left(P^\mu\gamma^\nu - P^\nu\gamma^\mu\right)\right] + \left[\frac{1}{4}g^{\mu\nu}\not{P}\right] \quad . \end{aligned} \tag{A1}$$

iv) The operator $P^\alpha P^\beta\gamma^\gamma$ is decomposed into spin-3, spin-2, spin-1 and spin-0 parts. There is one ambiguity in the definition of the spin-0 part: Sandwiched between nucleon states, it can be transformed into a spin-1 operator via the equation of motion (see Eqns. (33-35)).

$$P^\alpha P^\beta\gamma^\gamma = \left[P^\alpha P^\beta\gamma^\gamma\right]_{\text{Spin}-3} + \left[P^\alpha P^\beta\gamma^\gamma\right]_{\text{Spin}-2} + \left[P^\alpha P^\beta\gamma^\gamma\right]_{\text{Spin}-1} + \left[P^\alpha P^\beta\gamma^\gamma\right]_{\text{Spin}-0}$$

$$
\begin{aligned}
&= \Bigg[\frac{1}{6} s^{\alpha\beta\gamma\delta} s_{\delta\sigma\tau\lambda} P^\sigma P^\tau \gamma^\lambda \\
&\quad -\frac{1}{18}\Big(g^{\alpha\beta}[P^2\gamma^\gamma + \not{P} P^\gamma + P^\gamma \not{P}] + g^{\alpha\gamma}[P^2\gamma^\beta + \not{P} P^\beta + P^\beta \not{P}] \\
&\qquad\qquad + g^{\gamma\beta}[P^2\gamma^\alpha + \not{P} P^\alpha + P^\alpha \not{P}]\Big)\Bigg] \\
&\quad + \Bigg[\frac{1}{3}\Big(2P^\alpha P^\beta \gamma^\gamma - P^\gamma P^\alpha \gamma^\beta - P^\beta P^\gamma \gamma^\alpha\Big) \\
&\quad -\frac{1}{9}\Big(g^{\alpha\beta}[2P^2\gamma^\gamma - \not{P} P^\gamma - P^\gamma \not{P}] + g^{\alpha\gamma}[2\not{P} P^\beta - P^\beta \not{P} - P^2\gamma^\beta] \\
&\qquad\qquad + g^{\gamma\beta}[2P^\alpha \not{P} - \not{P} P^\alpha - P^2\gamma^\alpha]\Big)\Bigg] \\
&\quad + \Bigg[\frac{1}{18}\Big(g^{\alpha\beta}[5P^2\gamma^\gamma - \not{P} P^\gamma - P^\gamma \not{P}] + g^{\alpha\gamma}[5\not{P} P^\beta - P^\beta \not{P} - P^2\gamma^\beta] \\
&\qquad\qquad + g^{\gamma\beta}[5P^\alpha \not{P} - \not{P} P^\alpha - P^2\gamma^\alpha]\Big)\Bigg] \\
&\quad + \Bigg[\frac{1}{6}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\delta\sigma\tau\lambda} P^\sigma P^\tau \gamma^\lambda \quad \Bigg] \quad . \qquad \text{(A2)}
\end{aligned}
$$

v) For the operator $\gamma^\alpha\gamma^\beta P^\gamma$ the situation is essentially the same as in iv), however, this operator has no spin-3 part due to the fact that different gamma matrices anticommute.

$$
\begin{aligned}
\gamma^\alpha\gamma^\beta P^\gamma &= \left[\gamma^\alpha\gamma^\beta P^\gamma\right]_{\text{Spin}-2} + \left[\gamma^\alpha\gamma^\beta P^\gamma\right]_{\text{Spin}-1} + \left[\gamma^\alpha\gamma^\beta P^\gamma\right]_{\text{Spin}-0} \\
&= \Bigg[\frac{1}{6}\Big(2\gamma^\alpha\gamma^\beta P^\gamma - \gamma^\beta\gamma^\gamma P^\alpha - \gamma^\gamma\gamma^\alpha P^\beta - 2\gamma^\beta\gamma^\alpha P^\gamma + \gamma^\gamma\gamma^\beta P^\alpha + \gamma^\alpha\gamma^\gamma P^\beta\Big) \\
&\quad -\frac{1}{6}\Big(g^{\alpha\gamma}[\not{P}\gamma^\beta - \gamma^\beta\not{P}] - g^{\beta\gamma}[\not{P}\gamma^\alpha - \gamma^\alpha\not{P}]\Big)\Bigg] \\
&\quad + \Bigg[\frac{1}{6}\Big(g^{\alpha\gamma}[\not{P}\gamma^\beta - \gamma^\beta\not{P}] - g^{\beta\gamma}[\not{P}\gamma^\alpha - \gamma^\alpha\not{P}]\Big) + g^{\alpha\beta}P^\gamma\Bigg] \\
&\quad + \Bigg[\frac{1}{6}\epsilon^{\alpha\beta\gamma\delta}\epsilon_{\delta\sigma\tau\lambda}\gamma^\sigma\gamma^\tau P^\lambda\Bigg] \quad . \qquad \text{(A3)}
\end{aligned}
$$

According to this decomposition we define the various matrix elements in the following manner, ordered according to the number of vector indices.

Scalar and pseudoscalar operators

$$\langle pS|\overline{q}_f(0)q_f(0)|pS\rangle = 2a_{f-}^{(0,3)}M \quad , \tag{A4}$$

$$\langle pS|\overline{q}_f(0)\gamma_5 q_f(0)|pS\rangle = 0 \quad . \tag{A5}$$

Vector and pseudovector operators

$$\langle pS|\overline{q}_f(0)P_\mu q_f(0)|pS\rangle = m_f\langle pS|\overline{q}_f(0)\gamma_\mu q_f(0)|pS\rangle \quad , \tag{A6}$$

$$\langle pS|\overline{q}_f(0)P_\mu\gamma_5 q_f(0)|pS\rangle = 0 \quad . \tag{A7}$$

The last equation was simplified using the equation of motion.

$$\langle pS|\overline{q}_f(0)\gamma_\mu q_f(0)|pS\rangle = 2a_f^{(0,2)}p_\mu \quad , \tag{A8}$$

$$\langle pS|\overline{q}_f(0)\gamma_\mu\gamma_5 q_f(0)|pS\rangle = 2a_{f5}^{(0,2)}S_\mu \quad . \tag{A9}$$

Rank-two tensor operators

In the spin-2 case the parametrization of the two possible operators is straightforward:

$$\langle pS|\overline{q}_f(0)\left[(P_\mu\gamma_\nu + P_\nu\gamma_\mu) - \frac{1}{2}g_{\mu\nu}\not{P}\right]q_f(0)|pS\rangle = 2a_f^{(1,2)}\left[2p_\mu p_\nu - \frac{1}{2}g_{\mu\nu}M^2\right] \ , \tag{A10}$$

$$\langle pS|\overline{q}_f(0)(P_\mu\gamma_\nu + P_\nu\gamma_\mu)\gamma_5 q_f(0)|pS\rangle = 2a_{f5}^{(1,2)}\left[p_\mu S_\nu + p_\nu S_\mu\right] \ . \tag{A11}$$

For spin-1 only the pseudo-tensor operator yields a non-zero matrix element:

$$\langle pS|\overline{q}_f(0)(P_\mu\gamma_\nu - P_\nu\gamma_\mu)q_f(0)|pS\rangle = 0 \ , \tag{A12}$$

$$\begin{aligned}\langle pS|\overline{q}_f(0)(P_\mu\gamma_\nu - P_\nu\gamma_\mu)\gamma_5 q_f(0)|pS\rangle &= m_f\langle pS|\overline{q}_f(0)(-i\sigma_{\mu\nu})\gamma_5 q_f(0)|pS\rangle \\ &= 2\frac{m_f}{M}a_{f5-}^{(0,2)}\left[p_\mu S_\nu - p_\nu S_\mu\right] \ . \end{aligned} \tag{A13}$$

$a_{f5-}^{(0,2)}$ is connected to the first moment of the parton distribution $h_1(x)$ which is proportional to the structure function $G_3(x)$.

Rank-three tensor operators

The operators $P_\alpha P_\beta \gamma_\sigma$ and $\gamma_\alpha \gamma_\beta P_\sigma$ have to be considered.

$\gamma_\alpha \gamma_\beta P_\sigma$**: Spin-2**

Again, only the operator with γ_5 has a nonvanishing matrix element, i.e.:

$$\langle pS|\,\overline{q}_f(0)\gamma_\alpha\gamma_\beta P_\sigma\Big|_{\mathrm{spin-2}}\,q_f(0)|pS\rangle = 0 \quad , \tag{A14}$$

$$\begin{aligned}
&\langle pS|\,\overline{q}_f(0)\gamma_\alpha\gamma_\beta P_\sigma\Big|_{\mathrm{spin-2}}\,\gamma_5 q_f(0)|pS\rangle = \\
&= \langle pS|\overline{q}_f(0)\left[(-i)\sigma_{\alpha\beta}P^\sigma\gamma_5 - \frac{1}{6}\left[g_{\alpha\sigma}(\not{P}\gamma_\beta - \gamma_\beta\not{P}) - g_{\beta\sigma}(\not{P}\gamma_\alpha - \gamma_\alpha\not{P})\right]\gamma_5\right]q_f(0)|pS\rangle \\
&= \frac{2}{M}a_{f5-}^{(1,2)}\left[(p_\alpha S_\beta - p_\beta S_\alpha)p_\sigma - \frac{M^2}{3}[g_{\alpha\sigma}S_\beta - g_{\beta\sigma}S_\alpha]\right] \quad .
\end{aligned} \tag{A15}$$

$a_{f5-}^{(1,2)}$ is connected to $h_1(x)$ as indicated by $(-i)\sigma_{\alpha\beta}$.

$\gamma_\alpha \gamma_\beta P_\sigma$**: Spin-0**

By means of the equation of motion we get:

$$\begin{aligned}
\frac{1}{6}\langle pS|\overline{q}_f(0)\epsilon_{\alpha\beta\sigma\delta}\epsilon^\delta_{\ \tau\rho\lambda}\gamma^\tau\gamma^\rho P^\lambda q_f(0)|pS\rangle &= \frac{1}{3}im_f\epsilon_{\alpha\beta\sigma\delta}\langle pS|\overline{q}_f(0)\gamma^\delta\gamma_5 q_f(0)|pS\rangle \\
&= \frac{2}{3}im_f\epsilon_{\alpha\beta\sigma\delta}a_{f5}^{(0,2)}S^\delta \quad ,
\end{aligned} \tag{A16}$$

$$\frac{1}{6}\langle pS|\overline{q}_f(0)\epsilon_{\alpha\beta\sigma\delta}\epsilon^\delta_{\ \tau\rho\lambda}\gamma^\tau\gamma^\rho P^\lambda\gamma_5 q_f(0)|pS\rangle = 0 \quad . \tag{A17}$$

$P_\alpha P_\beta \gamma_\sigma$**: Spin-3**

The parametrization of the spin-3 part is straightforward:

$$\begin{aligned}
&\langle pS|\,\overline{q}_f(0)P_\alpha P_\beta\gamma_\sigma\Big|_{\mathrm{spin-3}}\,q_f(0)|pS\rangle \\
&= 2a_f^{(2,2)}\left[p_\alpha p_\beta p_\sigma - \frac{M^2}{6}[g_{\alpha\beta}p_\sigma + g_{\beta\sigma}p_\alpha + g_{\alpha\sigma}p_\beta]\right] \quad ,
\end{aligned} \tag{A18}$$

$$\langle pS|\,\overline{q}_f(0)P_\alpha P_\beta\gamma_\sigma\Big|_{\mathrm{spin-3}}\,\gamma_5 q_f(0)|pS\rangle$$

$$= 2a_{f5}^{(2,2)} \left[\frac{1}{3}(p_\alpha p_\beta S_\sigma + p_\beta p_\sigma S_\alpha + p_\sigma p_\alpha S_\beta) - \frac{M^2}{18}[g_{\alpha\beta}S_\sigma + g_{\beta\sigma}S_\alpha + g_{\alpha\sigma}S_\beta]\right] \quad . \tag{A19}$$

$P_\alpha P_\beta \gamma_\sigma$**: Spin-2**

In the spin-2 contribution the gluonic field strength tensor comes into play due to the following two identities:

$$g\tilde{G}^{\beta\sigma} = \Big\{ \not{P}(P^\beta\gamma^\sigma - P^\sigma\gamma^\beta) - (P^\beta\gamma^\sigma - P^\sigma\gamma^\beta)\not{P} + \frac{1}{2}\left(\not{P}[\gamma^\beta\gamma^\sigma - \gamma^\sigma\gamma^\beta]\not{P} - P^2[\gamma^\beta\gamma^\sigma - \gamma^\sigma\gamma^\beta]\right) + (P^\beta P^\sigma - P^\sigma P^\beta)\Big\}\gamma_5 \quad , \tag{A20}$$

and

$$\gamma^\alpha g\tilde{G}^{\beta\sigma} + \gamma^\beta g\tilde{G}^{\alpha\sigma} + \frac{1}{3}\left[2g^{\alpha\beta}g\tilde{G}^{\sigma\delta}\gamma_\delta - g^{\beta\sigma}g\tilde{G}^{\alpha\delta}\gamma_\delta - g^{\sigma\alpha}g\tilde{G}^{\beta\delta}\gamma_\delta\right]$$

$$= \Big\{ \Big(2P^\alpha P^\beta\gamma^\sigma + 2P^\beta P^\alpha\gamma^\sigma - P^\beta P^\sigma\gamma^\alpha - P^\sigma P^\beta\gamma^\alpha - P^\sigma P^\alpha\gamma^\beta - P^\alpha P^\sigma\gamma^\beta - \frac{2}{3}P^2\left[2g^{\alpha\beta}\gamma^\sigma - g^{\beta\sigma}\gamma^\alpha - g^{\sigma\alpha}\gamma^\beta\right]\Big)$$

$$-m\Big[\frac{1}{3}\left(2\gamma^\beta\gamma^\sigma P^\alpha - \gamma^\sigma\gamma^\alpha P^\beta - \gamma^\alpha\gamma^\beta P^\sigma - 2\gamma^\sigma\gamma^\beta P^\alpha + \gamma^\alpha\gamma^\sigma P^\beta + \gamma^\beta\gamma^\alpha P^\sigma\right) - \frac{1}{3}\left(g^{\alpha\beta}[\not{P}\gamma^\sigma - \gamma^\sigma\not{P}] - g^{\alpha\sigma}[\not{P}\gamma^\beta - \gamma^\beta\not{P}]\right) + \frac{1}{3}\left(2\gamma^\alpha\gamma^\sigma P^\beta - \gamma^\sigma\gamma^\beta P^\alpha - \gamma^\beta\gamma^\alpha P^\sigma - 2\gamma^\sigma\gamma^\alpha P^\beta + \gamma^\beta\gamma^\sigma P^\alpha + \gamma^\alpha\gamma^\beta P^\sigma\right) - \frac{1}{3}\left(g^{\beta\alpha}[\not{P}\gamma^\sigma - \gamma^\sigma\not{P}] - g^{\beta\sigma}[\not{P}\gamma^\alpha - \gamma^\alpha\not{P}]\right)\Big]\Big\}\gamma_5 \quad . \tag{A21}$$

Again, one of the two possible matrix elements is zero

$$\langle pS|\, \overline{q}_f(0)P_\alpha P_\beta\gamma_\sigma\Big|_{\mathrm{spin}-2}\, q_f(0)|pS\rangle$$

$$
\begin{aligned}
&= \langle pS|\overline{q}_f(0)\frac{1}{6}\Bigg[(\gamma_\alpha g\tilde{G}_{\beta\sigma} + \gamma_\beta g\tilde{G}_{\alpha\sigma})\gamma_5 \\
&\quad + \frac{1}{3}\left[2g_{\alpha\beta} g\tilde{G}_{\sigma\delta}\gamma^\delta - g_{\beta\sigma} g\tilde{G}_{\alpha\delta}\gamma^\delta - g_{\sigma\alpha} g\tilde{G}_{\beta\delta}\gamma^\delta\right]\gamma_5\Bigg] q_f(0)|pS\rangle \\
&= 0 \quad .
\end{aligned} \tag{A22}
$$

This implies that the gluonic matrix element is zero as well. In the other case we have for the gluonic representation and for the representation in terms of covariant derivatives:

$$
\begin{aligned}
&\langle pS|\overline{q}_f(0)\frac{1}{6}\Bigg[(\gamma_\alpha g\tilde{G}_{\beta\sigma} + \gamma_\beta g\tilde{G}_{\alpha\sigma}) \\
&+ \frac{1}{3}\left[2g_{\alpha\beta} g\tilde{G}_{\sigma\delta}\gamma^\delta - g_{\beta\sigma} g\tilde{G}_{\alpha\delta}\gamma^\delta - g_{\sigma\alpha} g\tilde{G}_{\beta\delta}\gamma^\delta\right]\Bigg] q_f(0)|pS\rangle \\
&= 2a_{f5}^{(2,3)}\left[\frac{1}{3}(2p_\alpha p_\beta S_\sigma - p_\beta p_\sigma S_\alpha - p_\sigma p_\alpha S_\beta) - \frac{M^2}{9}[2g_{\alpha\beta}S_\sigma - g_{\beta\sigma}S_\alpha - g_{\alpha\sigma}S_\beta]\right] \quad ,
\end{aligned} \tag{A23}
$$

and

$$
\begin{aligned}
&\langle pS|\,\overline{q}_f(0)P_\alpha P_\beta\gamma_\sigma\Big|_{\text{spin}-2}\gamma_5 q_f(0)|pS\rangle \\
&= 2\tilde{a}_{f5}^{(2,3)}\left[\frac{1}{3}(2p_\alpha p_\beta S_\sigma - p_\beta p_\sigma S_\alpha - p_\sigma p_\alpha S_\beta) - \frac{M^2}{9}[2g_{\alpha\beta}S_\sigma - g_{\beta\sigma}S_\alpha - g_{\alpha\sigma}S_\beta]\right] .
\end{aligned} \tag{A24}
$$

These two representations are equivalent except for mass terms. Explicitly one has the identity

$$
a_{f5}^{(2,3)} = \tilde{a}_{f5}^{(2,3)} - \frac{m_f}{M}a_{f5-}^{(1,2)} \quad . \tag{A25}
$$

$P_\alpha P_\beta \gamma_\sigma$: Spin-0

For the spin-0 contribution there exists also a transformation between a gluonic representation and a representation in form of covariant derivatives only:

$$\langle pS|\overline{q}_f(0)\epsilon_{\alpha\beta\sigma\delta}\epsilon^{\delta}{}_{\tau\rho\lambda}P^{\tau}P^{\rho}\gamma^{\lambda}q_f(0)|pS\rangle = i\langle pS|\overline{q}_f(0)\epsilon_{\alpha\beta\sigma\delta}g\tilde{G}^{\delta}{}_{\lambda}\gamma^{\lambda}q_f(0)|pS\rangle$$
$$= 2i\epsilon_{\alpha\beta\sigma\delta}M^2 a_{f5}^{(2,4)}S^{\delta}$$
$$= i\epsilon_{\alpha\beta\sigma\delta}\langle pS|\overline{q}_f(0)(P^2 - m_f^2)\gamma^{\delta}\gamma_5 q_f(0)|pS\rangle$$
$$= 2i\epsilon_{\alpha\beta\sigma\delta}\left[M^2\tilde{a}_{f5}^{(2,4)} - m_f^2 a_{f5}^{(0,2)}\right]S^{\delta}\ , \qquad \text{(A26)}$$

and analogously

$$\langle pS|\overline{q}_f(0)\epsilon_{\alpha\beta\sigma\delta}\epsilon^{\delta}{}_{\tau\rho\lambda}P^{\tau}P^{\rho}\gamma^{\lambda}\gamma_5 q_f(0)|pS\rangle = i\langle pS|\overline{q}_f(0)\epsilon_{\alpha\beta\sigma\delta}g\tilde{G}^{\delta}{}_{\lambda}\gamma^{\lambda}\gamma_5 q_f(0)|pS\rangle$$
$$= 2i\epsilon_{\alpha\beta\sigma\delta}M^2 a_f^{(2,4)}P^{\delta}$$
$$= i\epsilon_{\alpha\beta\sigma\delta}\langle pS|\overline{q}_f(0)(P^2 - m_f^2)\gamma^{\delta}q_f(0)|pS\rangle$$
$$= 2i\epsilon_{\alpha\beta\sigma\delta}\left[M^2\tilde{a}_f^{(2,4)} - m_f^2 a_f^{(0,2)}\right]P^{\delta}\ . \qquad \text{(A27)}$$

Obviously

$$M^2 a_{f5}^{(2,4)} = M^2\tilde{a}_{f5}^{(2,4)} - m_f^2 a_{f5}^{(0,2)} \quad , \qquad \text{(A28)}$$
$$M^2 a_f^{(2,4)} = M^2\tilde{a}_f^{(2,4)} - m_f^2 a_f^{(0,2)} \quad . \qquad \text{(A29)}$$

APPENDIX B: OPERATORS GENERATED BY THE CAT EAR DIAGRAM

In the required approximation the cat ear diagram (fig.4) is not connected with the external gluonic field, so it is sufficient to analyze ordinary Feynman diagrams in zeroth order in $(\not{P} - m_f)/Q$. In the case of γ or Z^0 exchange the corresponding virtual forward Compton scattering amplitude then is given by

$$T_{\mu\nu}^{(\text{cat ear }\gamma,Z^0)}\Big|_{0.\ order} = \sum_{ff'}\frac{g^2}{2!}i^3\int d^4\xi e^{iq\xi}\int dz_1 dz_2\langle pS|\Big\{\overline{q}_f(\xi)\gamma_\mu(V_f + A_f\gamma_5)iS_f(\xi,z_1)\gamma^{\rho}t^a q_f(z_1)$$
$$+\overline{q}_f(z_1)\gamma^{\rho}t^a iS_f(z_1,\xi)\gamma_\mu(V_f + A_f\gamma_5)q_f(\xi)\Big\}iD^{ab}_{\rho\sigma}(z_1,z_2)$$
$$\times\Big\{\overline{q}_{f'}(z_2)\gamma^{\sigma}t^b iS_{f'}(z_2,0)\gamma_\nu(V_{f'} + A_{f'}\gamma_5)q_{f'}(0)$$
$$+\overline{q}_{f'}(0)\gamma_\nu(V_{f'} + A_{f'}\gamma_5)iS_{f'}(0,z_2)\gamma^{\sigma}t^b q_{f'}(z_2)\Big\}|pS\rangle \quad , \qquad \text{(B1)}$$

where we have to insert the free massless fermion-propagator. $D^{ab}_{\rho\sigma}(z_1, z_2)$ is the gluonic propagator connecting the "ears" of the cat ear diagram. t_a, $a = 1, \ldots, 8$ denote the color matrices ($\mathrm{tr}(\mathrm{t^a t^b}) = \delta^{\mathrm{ab}}/2$). In that way we obtain

$$
\begin{aligned}
T_{\mu\nu}^{(\text{cat ear } \mathrm{Z}^0,\gamma)}\Big|_{0.\ order} = -\frac{g^2}{2}\sum_{ff'}\langle pS|\Big\{&\overline{q}_f(0)\gamma_\mu(V_f + A_f\gamma_5)\frac{1}{\not{q}}\gamma_\rho t^a q_f(0) \\
&-\overline{q}_f(0)\gamma_\rho t^a\frac{1}{\not{q}}\gamma_\mu(V_f + A_f\gamma_5)q_f(0)\Big\}\frac{-g^{\sigma\rho}}{q^2} \\
&\times\Big\{\overline{q}_{f'}(0)\gamma_\sigma t^a\frac{1}{\not{q}}\gamma_\nu(V_{f'} + A_{f'}\gamma_5)q_{f'}(0) \\
&-\overline{q}_{f'}(0)\gamma_\nu(V_{f'} + A_{f'}\gamma_5)\frac{1}{\not{q}}\gamma_\sigma t^a q_{f'}(0)\Big\}|pS\rangle \quad . \qquad \text{(B2)}
\end{aligned}
$$

The corresponding expression for the W^- exchange is

$$
\begin{aligned}
T_{\mu\nu}^{(\text{cat ear } \mathrm{W}^-)}\Big|_{0.\ order} = -\frac{g^2}{2}\langle pS|\Big\{&\overline{u}(0)\gamma_\mu(V + A\gamma_5)\frac{1}{\not{q}}\gamma_\rho t^a d(0) \\
&-\overline{u}(0)\gamma_\rho t^a\frac{1}{\not{q}}\gamma_\mu(V + A\gamma_5)d(0)\Big\}\frac{-g^{\sigma\rho}}{q^2} \\
&\times\Big\{\overline{d}(0)\gamma_\sigma t^a\frac{1}{\not{q}}\gamma_\nu(V + A\gamma_5)u(0) \\
&-\overline{d}(0)\gamma_\nu(V + A\gamma_5)\frac{1}{\not{q}}\gamma_\sigma t^a u(0)\Big\}|pS\rangle \\
&+ \text{further flavor combinations} \quad . \qquad \text{(B3)}
\end{aligned}
$$

(The case of W^+ exchange is obtained by exchanging the quark flavors.) Introducing reduced matrix elements of the occurring four-quark operators we finally get in zeroth order

$$
\begin{aligned}
T_{\mu\nu}^{(\text{cat ear } \mathrm{Z}^0,\gamma)}\Big|_{0.\ \text{order}} = g^2\Big\{&\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\left[\frac{\omega^2}{Q^2}C^{(2)}_{=} + 2\frac{M^2}{Q^4}\left[C^{(0)}_{=} + C^{(2)}_{=}\right]\right] \\
&+\frac{Sq}{\nu}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\left[\frac{\omega^2}{Q^2}C^{(2)}_{\times}\right] \\
&+\frac{1}{\nu}\hat{p}_\mu\hat{p}_\nu\left[2\frac{\omega}{Q^2}C^{(2)}_{=}\right] + \frac{1}{2\nu}(\hat{S}_\mu\hat{p}_\nu + \hat{S}_\nu\hat{p}_\mu)\left[2\frac{\omega}{Q^2}C^{(2)}_{\times}\right] \quad ,
\end{aligned}
$$

$$T_{\mu\nu}^{(\text{cat ear W}^-)}\Big|_{0.\text{ order}} = g^2\Bigg\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\left[\frac{\omega^2}{Q^2}C^{(2)}_{ud=} + 2\frac{M^2}{Q^4}\left[C^{(0)}_{ud=} + C^{(2)}_{ud=}\right]\right]$$

$$+\frac{Sq}{\nu}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)\left[\frac{\omega^2}{Q^2}C^{(2)}_{ud\times}\right]$$

$$+\frac{1}{\nu}\hat{p}_\mu\hat{p}_\nu\left[2\frac{\omega}{Q^2}C^{(2)}_{ud=}\right] + \frac{1}{2\nu}(\hat{S}_\mu\hat{p}_\nu + \hat{S}_\nu\hat{p}_\mu)\left[2\frac{\omega}{Q^2}C^{(2)}_{ud\times}\right] \quad , \qquad \text{(B4)}$$

where only the expression for the $u \to d$ flavor combination is given. Here the flavor independent matrix elements $C^{(j)}_{=/\times}$ are defined by

$$C^{(j)}_{=/\times} := \sum_f C^{(j)}_{ff=/\times}, \qquad j = 0, 1, 2, \ldots \quad . \qquad \text{(B5)}$$

Note, that the cat ear diagram (B4) is symmetric in μ and ν. Therefore in zeroth order of $\not{P} - m_f$ it gives only twist-4 and twist-6 contributions to structure functions associated with symmetric Lorentz structures, such as F_1 and F_2. Since in the required approximation quark masses do not occur, the cat ear does not contribute to structure functions given by non-conserved currents. Therefore we find that cat-ear diagrams contribute only to the structure functions F_1, F_2, a_1, and b_2.

For the definition of the reduced matrix elements of the cat ear diagram we need the following definition of four-quark operators:

$$\hat{\mathcal{O}}^{\sigma\sigma'}_{55,ff'} := \overline{q}_f(0)\gamma^\sigma\gamma_5 t^a q_{f'}(0)\overline{q}_{f'}(0)\gamma^{\sigma'}\gamma_5 t^a q_f(0) A_f A_{f'} \quad , \qquad \text{(B6)}$$

$$\hat{\mathcal{O}}^{\sigma\sigma'}_{5X,ff'} := \overline{q}_f(0)\gamma^\sigma\gamma_5 t^a q_{f'}(0)\overline{q}_{f'}(0)\gamma^{\sigma'} t^a q_f(0) A_f V_{f'} \quad , \qquad \text{(B7)}$$

$$\hat{\mathcal{O}}^{\sigma\sigma'}_{X5,ff'} := \overline{q}_f(0)\gamma^\sigma t^a q_{f'}(0)\overline{q}_{f'}(0)\gamma^{\sigma'}\gamma_5 t^a q_f(0) V_f A_{f'} \quad , \qquad \text{(B8)}$$

$$\hat{\mathcal{O}}^{\sigma\sigma'}_{XX,ff'} := \overline{q}_f(0)\gamma^\sigma t^a q_{f'}(0)\overline{q}_{f'}(0)\gamma^{\sigma'} t^a q_f(0) V_f V_{f'} \quad . \qquad \text{(B9)}$$

Note that for γ and Z^0 exchange $f = f'$, and in the case of $W^\pm$ exchange we always

have $V_f = V_{f'} = V$ and $A_f = A_{f'} = A$. We then define the reduced matrix elements to be

$$2C^{(2)}_{ff'=}\left(p^\sigma p^{\sigma'} - \frac{M^2}{4}g^{\sigma\sigma'}\right) = \langle pS|\frac{1}{2}(\hat{\mathcal{O}}^{\sigma\sigma'}_{55,ff'} + \hat{\mathcal{O}}^{\sigma\sigma'}_{XX,ff'} + \hat{\mathcal{O}}^{\sigma'\sigma}_{55,ff'} + \hat{\mathcal{O}}^{\sigma'\sigma}_{XX,ff'}) \\ -\frac{1}{4}g^{\sigma\sigma'}g_{\alpha\alpha'}(\hat{\mathcal{O}}^{\alpha\alpha'}_{55,ff'} + \hat{\mathcal{O}}^{\alpha\alpha'}_{XX,ff'})|pS\rangle \quad , \tag{B10}$$

$$0 = \langle pS|\frac{1}{2}(\hat{\mathcal{O}}^{\sigma\sigma'}_{55,ff'} + \hat{\mathcal{O}}^{\sigma\sigma'}_{XX,ff'} - \hat{\mathcal{O}}^{\sigma'\sigma}_{55,ff'} - \hat{\mathcal{O}}^{\sigma'\sigma}_{XX,ff'})|pS\rangle \quad , \tag{B11}$$

$$2M^2C^{(0)}_{ff'=} = \langle pS|g_{\alpha\alpha'}(\hat{\mathcal{O}}^{\alpha\alpha'}_{55,ff'} + \hat{\mathcal{O}}^{\alpha\alpha'}_{XX,ff'})|pS\rangle \quad , \tag{B12}$$

$$C^{(2)}_{ff'\times}\left(S^\sigma p^{\sigma'} + S^{\sigma'}p^\sigma\right) = \langle pS|\frac{1}{2}(\hat{\mathcal{O}}^{\sigma\sigma'}_{5X,ff'} + \hat{\mathcal{O}}^{\sigma\sigma'}_{X5,ff'} + \hat{\mathcal{O}}^{\sigma'\sigma}_{5X,ff'} + \hat{\mathcal{O}}^{\sigma'\sigma}_{X5,ff'}) \\ -\frac{1}{4}g^{\sigma\sigma'}g_{\alpha\alpha'}(\hat{\mathcal{O}}^{\alpha\alpha'}_{5X,ff'} + \hat{\mathcal{O}}^{\alpha\alpha'}_{X5,ff'})|pS\rangle \quad , \tag{B13}$$

$$0 = \langle pS|\frac{1}{2}(\hat{\mathcal{O}}^{\sigma\sigma'}_{5X,ff'} + \hat{\mathcal{O}}^{\sigma\sigma'}_{X5,ff'} - \hat{\mathcal{O}}^{\sigma'\sigma}_{5X,ff'} - \hat{\mathcal{O}}^{\sigma'\sigma}_{X5,ff'}) \quad , \tag{B14}$$

$$0 = \langle pS|g_{\alpha\alpha'}(\hat{\mathcal{O}}^{\alpha\alpha'}_{5X,ff'} + \hat{\mathcal{O}}^{\alpha\alpha'}_{X5,ff'})|pS\rangle \quad . \tag{B15}$$

Note, that the operators $\hat{\mathcal{O}}_{XX,ff'}, \hat{\mathcal{O}}_{5X,ff'}, \hat{\mathcal{O}}_{X5,ff'}$ can be transformed by means of the equation of motion

$$D^\alpha G^a_{\alpha\beta} = -\sum_f g\overline{q}_f\gamma_\beta t^a q_f, \tag{B16}$$

where D_α is the covariant QCD derivative and $G^a_{\alpha\beta}$ the gluonic field tensor. However, as this operation cannot be applied to $\hat{\mathcal{O}}_{55,ff'}$, we did not perform it to keep a uniform notation.

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Figure Captions

Figure 1: Kinematic variables in deep inelastic scattering: An electron with four-momentum k and spin σ is scattered by a nucleon with four-momentum p and polarization λ through the exchange of a gauge boson with four-momentum q. The nucleon fragments into a variety of hadrons X, which together carry the four-momentum p_X.

Figure 2: The contour of integration of the virtual forward Compton scattering amplitude $T_{\mu\nu}$. The regions where the function $T_{\mu\nu}(\omega)$ has singularities are shadowed. The four big arrows show the integration path. It can be completed by the outer circle in the infinity thus being equivalent to the inner circle.

Figure 3: The hand bag diagram: It represents the virtual Compton forward scattering amplitude, for quark (solid line) gauge boson (sinuous line) scattering.

Figure 4: The cat ear diagram: The lines represent: Gauge boson (sinuous line), gluon (curly line), quark (straight line).

TABLES

	unpolarized structure functions	polarized structure functions
parity-conserved:		
current conserved	F_1, F_2	g_1, g_2
current non-conserved	F_4, F_5	g_3
parity-violating:		
current conserved	F_3	a_1, a_2, b_1
current non-conserved		a_4, a_5, b_2

TABLE I. The 14 nucleon structure functions.

Moments of structure functions		
$\int_0^1 xdxF_1^{(\gamma,Z^0)}(x,Q^2)$	$=$	$\sum_f \frac{1}{2}a_f^{(1,2)}\left(V_f^2+A_f^2\right)+\mathcal{O}(1/Q^2)$
$\int_0^1 dxF_2^{(\gamma,Z^0)}(x,Q^2)$	$=$	$\sum_f a_f^{(1,2)}\left(V_f^2+A_f^2\right)+\mathcal{O}(1/Q^2)$
$\int_0^1 dxF_3^{(\gamma,Z^0)}(x,Q^2)$	$=$	$\sum_f \left\{a_f^{(0,2)}+\frac{2}{9}\frac{M^2}{Q^2}\left[3a_f^{(2,2)}+8a_f^{(2,4)}\right]\right.$
		$\left.-\frac{2}{3}\frac{m_f^2}{Q^2}a_f^{(0,2)}\right\}2V_fA_f+\mathcal{O}(1/Q^4)$
$\int_0^1 x^2dxF_3^{(\gamma,Z^0)}(x,Q^2)$	$=$	$\sum_f a_f^{(2,2)}2V_fA_f+\mathcal{O}(1/Q^2)$
$\int_0^1 dxF_4^{(\gamma,Z^0)}(x,Q^2)$	$=$	$\sum_f \frac{m_fM}{Q^2}a_{f-}^{(0,3)}A_f^2+\mathcal{O}(1/Q^4)$
$\int_0^1 x^2dxF_4^{(\gamma,Z^0)}(x,Q^2)$	$=$	$0+\mathcal{O}(1/Q^2)$
$\int_0^1 xdxF_5^{(\gamma,Z^0)}(x,Q^2)$	$=$	$0+\mathcal{O}(1/Q^2)$
$\int_0^1 dxF_1^{(\overline{\nu}-\nu)}(x,Q^2)$	$=$	$a_{\Delta V}^{(0,2)}+\frac{8}{9}\frac{M^2}{Q^2}[3a_{\Delta V}^{(2,2)}-a_{\Delta V}^{(2,4)}]$
		$+\frac{1}{3}\frac{1}{Q^2}[m_{\Delta V}^2-3{m'}_{\Delta V}^2]a_{\Delta V}^{(0,2)}+\mathcal{O}(1/Q^4)$
$\int_0^1 xdxF_1^{(\overline{\nu}+\nu)}(x,Q^2)$	$=$	$a_S^{(1,2)}+\mathcal{O}(1/Q^2)$
$\int_0^1 x^2dxF_1^{(\overline{\nu}-\nu)}(x,Q^2)$	$=$	$a_{\Delta V}^{(2,2)}+\mathcal{O}(1/Q^2)$
$\int_0^1 dxF_2^{(\overline{\nu}+\nu)}(x,Q^2)$	$=$	$2a_S^{(1,2)}+\mathcal{O}(1/Q^2)$
$\int_0^1 xdxF_2^{(\overline{\nu}-\nu)}(x,Q^2)$	$=$	$2a_{\Delta V}^{(2,2)}+\mathcal{O}(1/Q^2)$
$\int_0^1 dxF_3^{(\overline{\nu}+\nu)}(x,Q^2)$	$=$	$-2a_V^{(0,2)}-\frac{4}{9}\frac{M^2}{Q^2}[3a_V^{(2,2)}+8a_V^{(2,4)}]$
		$-\frac{2}{3}\frac{1}{Q^2}[m_V^2-3{m'}_V^2]a_V^{(0,2)}+\mathcal{O}(1/Q^4)$
$\int_0^1 xdxF_3^{(\overline{\nu}-\nu)}(x,Q^2)$	$=$	$-2a_{\Delta S}^{(1,2)}+\mathcal{O}(1/Q^2)$

$$\int_0^1 x^2 dx F_3^{(\overline{\nu}+\nu)}(x,Q^2) \quad = \quad -2a_V^{(2,2)} + \mathcal{O}(1/Q^2)$$

$$\begin{aligned}
\int_0^1 dx F_4^{(\overline{\nu}+\nu)}(x,Q^2) &= \frac{m_S M}{Q^2} a_{S-}^{(0,3)} + \mathcal{O}(1/Q^4) \\
\int_0^1 x dx F_4^{(\overline{\nu}-\nu)}(x,Q^2) &= +\frac{1}{2}\frac{1}{Q^2}[3m_{\Delta V}^2 - m'^2_{\Delta V}] a_{\Delta V}^{(0,2)} + \mathcal{O}(1/Q^4) \\
\int x^2 dx F_4^{(\overline{\nu}+\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^2) \\
\int x^3 dx F_4^{(\overline{\nu}-\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx F_5^{(\overline{\nu}-\nu)}(x,Q^2) &= +2\frac{1}{Q^2}[m_{\Delta V}^2 - m'^2_{\Delta V}] a_{\Delta V}^{(0,2)} + \mathcal{O}(1/Q^4) \\
\int_0^1 x dx F_5^{(\overline{\nu}+\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^2) \\
\int_0^1 x^2 dx F_5^{(\overline{\nu}-\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx g_1^{(\gamma,Z^0)}(x,Q^2) &= \sum_f \Bigg[\left\{ \frac{1}{2} a_{f5}^{(0,2)} + \frac{1}{9}\frac{M^2}{Q^2}\left(a_{f5}^{(2,2)} + 4a_{f5}^{(2,3)} + 4a_{f5}^{(2,4)} + 4\frac{m_f}{M} a_{f5-}^{(1,2)} \right) \right. \\
&\qquad \left. -\frac{2}{9}\frac{m_f^2}{Q^2} a_{f5}^{(0,2)} \right\} (V_f^2 + A_f^2) \\
&\qquad - \left\{ \frac{1}{3}\frac{m_f^2}{Q^2} a_{f5}^{(0,2)} \right\} (V_f^2 - A_f^2) \Bigg] + \mathcal{O}(1/Q^4) \\
\int_0^1 x^2 dx g_1^{(\gamma,Z^0)}(x,Q^2) &= \sum_f \frac{1}{2} a_{f5}^{(2,2)} (V_f^2 + A_f^2) + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx g_2^{(\gamma,Z^0)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^4) \\
\int_0^1 x^2 dx g_2^{(\gamma,Z^0)}(x,Q^2) &= -\sum_f \frac{1}{3} \left\{ a_{f5}^{(2,2)} - a_{f5}^{(2,3)} - \frac{m_f}{M} a_{f5-}^{(1,2)} \right\} (V_f^2 + A_f^2) + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\int_0^1 x dx g_3^{(\gamma,Z^0)}(x,Q^2) \quad = \quad 0 + \mathcal{O}(1/Q^2)$$

$$\begin{aligned}
\int_0^1 dx g_1^{(\overline{\nu}+\nu)}(x,Q^2) &= a_{S5}^{(0,2)} + \frac{2}{9}\frac{M^2}{Q^2}\left(a_{S5}^{(2,2)} + 4a_{S5}^{(2,3)} + 4a_{S5}^{(2,4)} + 4\frac{m_S}{M} a_{S5-}^{(1,2)} \right) \\
&\quad + \frac{1}{9}\frac{1}{Q^2}[5m_S^2 - 9m'^2_S] a_{S5}^{(0,2)} + \mathcal{O}(1/Q^4) \\
\int_0^1 x dx g_1^{(\overline{\nu}-\nu)}(x,Q^2) &= a_{\Delta V 5}^{(1,2)} + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\int_0^1 x^2 dx g_1^{(\overline{\nu}+\nu)}(x,Q^2) = a_{S5}^{(2,2)} + \mathcal{O}(1/Q^2)$$

$$\begin{aligned}
\int_0^1 dx g_2^{(\overline{\nu}+\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^4) \\
\int_0^1 x dx g_2^{(\overline{\nu}-\nu)}(x,Q^2) &= -\frac{1}{2}\left(a_{\Delta V5}^{(1,2)} - \frac{m_{\Delta V}}{M} a_{\Delta V5-}^{(0,2)}\right) + \mathcal{O}(1/Q^2) \\
\int_0^1 x^2 dx g_2^{(\overline{\nu}+\nu)}(x,Q^2) &= -\frac{2}{3}\left[a_{S5}^{(2,2)} - a_{S5}^{(2,3)} - \frac{m_S}{M} a_{S5-}^{(1,2)}\right] + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx g_3^{(\overline{\nu}-\nu)}(x,Q^2) &= \frac{m_{\Delta V}}{M} a_{\Delta V5-}^{(0,2)} + \mathcal{O}(1/Q^2) \\
\int_0^1 x dx g_3^{(\overline{\nu}+\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 x dx a_1^{(\gamma,Z^0)}(x,Q^2) &= \sum_f a_{f5}^{(1,2)} A_f V_f + \mathcal{O}(1/Q^2) \\
\int_0^1 dx a_2^{(\gamma,Z^0)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^2) \\
\int_0^1 x^2 dx a_4^{(\gamma,Z^0)}(x,Q^2) &= -\sum_f \frac{1}{2}\frac{m_f}{M} a_{f5-}^{(0,2)} A_f V_f + \mathcal{O}(1/Q^2) \\
\int_0^1 x dx a_5^{(\gamma,Z^0)}(x,Q^2) &= -\sum_f \frac{m_f}{M} a_{f5-}^{(0,2)} A_f V_f + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx b_1^{(\gamma,Z^0)}(x,Q^2) &= 2\sum_f (a_{f5}^{(1,2)} + \frac{m_f}{M} a_{f5-}^{(0,2)}) A_f V_f + \mathcal{O}(1/Q^2) \\
\int_0^1 dx b_2^{(\gamma,Z^0)}(x,Q^2) &= -2\sum_f \frac{m_f}{M} a_{f5-}^{(0,2)} A_f V_f + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\begin{aligned}
\int_0^1 dx a_1^{(\overline{\nu}-\nu)}(x,Q^2) &= -a_{\Delta S5}^{(0,2)} - \frac{8}{9}\frac{M^2}{Q^2}[a_{\Delta S5}^{(2,2)} - 2a_{\Delta S5}^{(2,3)} + a_{\Delta S5}^{(2,4)}] + \frac{4}{9}\frac{m_{\Delta S} M}{Q^2} a_{\Delta S5-}^{(1,2)} \\
&\quad -\frac{1}{9}\frac{1}{Q^2}[5m_{\Delta S}^2 - 9m'^2_{\Delta S}] a_{\Delta S5}^{(0,2)} + \mathcal{O}(1/Q^4) \\
\int_0^1 x dx a_1^{(\overline{\nu}+\nu)}(x,Q^2) &= -a_{V5}^{(1,2)} + \mathcal{O}(1/Q^2) \\
\int_0^1 x^2 dx a_1^{(\overline{\nu}-\nu)}(x,Q^2) &= -a_{\Delta S5}^{(2,2)} + \mathcal{O}(1/Q^2)
\end{aligned}$$

$$\int_0^1 dx a_2^{(\overline{\nu}+\nu)}(x,Q^2) = 0 + \mathcal{O}(1/Q^2)$$

$$
\begin{aligned}
\int_0^1 xdxa_2^{(\overline{\nu}-\nu)}(x,Q^2) &= -\tfrac{2}{3}a_{\Delta S5}^{(2,2)} + \tfrac{8}{3}a_{\Delta S5}^{(2,3)} + \tfrac{2}{3}\tfrac{m_{\Delta S}}{M}a_{\Delta S5-}^{(1,2)} + \mathcal{O}(1/Q^2)
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 xdxa_4^{(\overline{\nu}-\nu)}(x,Q^2) &= +\tfrac{1}{2}\tfrac{1}{Q^2}[m_{\Delta S}^2 - m'^2_{\Delta S}]a_{\Delta S5}^{(0,2)} + \mathcal{O}(1/Q^4) \\
\int_0^1 x^2dxa_4^{(\overline{\nu}+\nu)}(x,Q^2) &= \tfrac{1}{2}\tfrac{m_V}{M}a_{V5-}^{(0,2)} + \mathcal{O}(1/Q^2) \\
\int_0^1 x^3dxa_4^{(\overline{\nu}-\nu)}(x,Q^2) &= \tfrac{1}{2}\tfrac{m_{\Delta S}}{M}a_{\Delta S5-}^{(1,2)} + \mathcal{O}(1/Q^2)
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 dxa_5^{(\overline{\nu}-\nu)}(x,Q^2) &= 0 + \mathcal{O}(1/Q^4) \\
\int_0^1 xdxa_5^{(\overline{\nu}+\nu)}(x,Q^2) &= \tfrac{m_V}{M}a_{V5-}^{(0,2)} + \mathcal{O}(1/Q^2) \\
\int_0^1 x^2dxa_5^{(\overline{\nu}-\nu)}(x,Q^2) &= \tfrac{m_{\Delta S}}{M}a_{\Delta S5-}^{(1,2)} + \mathcal{O}(1/Q^2)
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 dxb_1^{(\overline{\nu}+\nu)}(x,Q^2) &= -2\left(a_{V5}^{(1,2)} + \tfrac{m_V}{M}a_{V5-}^{(0,2)}\right) + \mathcal{O}(1/Q^2) \\
\int_0^1 xdxb_1^{(\overline{\nu}-\nu)}(x,Q^2) &= -\left[\tfrac{4}{3}a_{\Delta S5}^{(2,2)} + \tfrac{8}{3}a_{\Delta S5}^{(2,3)} + \tfrac{8}{3}\tfrac{m_{\Delta S}}{M}a_{\Delta S5-}^{(1,2)}\right] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

$$
\begin{aligned}
\int_0^1 dxb_2^{(\overline{\nu}+\nu)}(x,Q^2) &= 2\tfrac{m_V}{M}a_{V5-}^{(0,2)} + \mathcal{O}(1/Q^2) \\
\int_0^1 xdxb_2^{(\overline{\nu}-\nu)}(x,Q^2) &= 2\tfrac{m_{\Delta S}}{M}a_{\Delta S5-}^{(1,2)} + \mathcal{O}(1/Q^2)
\end{aligned}
$$

TABLE II. Moments of structure functions with calculated Q^2 corrections. Both the neutrino and the γ/Z^0 case are given.

	sum rule	reference
unpolarized		
	$\int_0^1 dx(F_1^{\overline{\nu}p} - F_1^{\nu p}) = 1\eta_W$	Bjorken (1967) [7]
	$\int_0^1 \frac{dx}{x}(F_2^{\overline{\nu}p} - F_2^{\nu p}) = 2\eta_W$	Adler (1966) [27]
	$\int_0^1 dx(F_3^{\overline{\nu}p} + F_3^{\nu p}) = -6\eta_W$	Gross, Llewellyn-Smith (1969) [8]
polarized		
	$\int_0^1 dx(g_1^p(x) - g_1^n(x)) = \frac{C}{2}\frac{g_A}{g_V}$	Bjorken (1966) [5]
	$\int_0^1 dx g_1^p(x) = \frac{1}{18}\frac{g_A}{g_V}\frac{9F/D-1}{F/D+1}$	Ellis, Jaffe (1974) [31]
	$\int_0^1 dx a_1^{(\overline{\nu}-\nu)(p-n)} = -2\frac{g_A}{g_V}\eta_W$	Wray (1972) [20]

TABLE III. Sum rules in the Bjorken limit ($Q^2 \to \infty$).

	relation	reference
unpolarized		
	$F_2(x) = 2xF_1(x)$	Callan, Gross (1969) [26]
polarized		
	$g_2(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$	
	$+ \begin{array}{c} \text{twist} - 3 \text{ contributions} \\ \text{and mass corrections} \end{array}$	Wandzura,Wilczek (1977) [29]
	$b_2(x) = 2xa_5(x) = 4x^2a_4(x)$	
	$2xa_1(x) = a_2(x) + b_1(x) + b_2(x)$	Dicus (1972) [19]

TABLE IV. Relations between structure functions in the Bjorken limit ($Q^2 \to \infty$).

	parity-conserved	parity-violating
amplitude $\overline{\nu} + \nu$	$(singlet)\ a_S$	$(valence)\ a_V$
amplitude $\overline{\nu} - \nu$	$a_{\Delta V}$	$a_{\Delta S}$

TABLE V. The four possible flavor combinations for neutrino scattering.

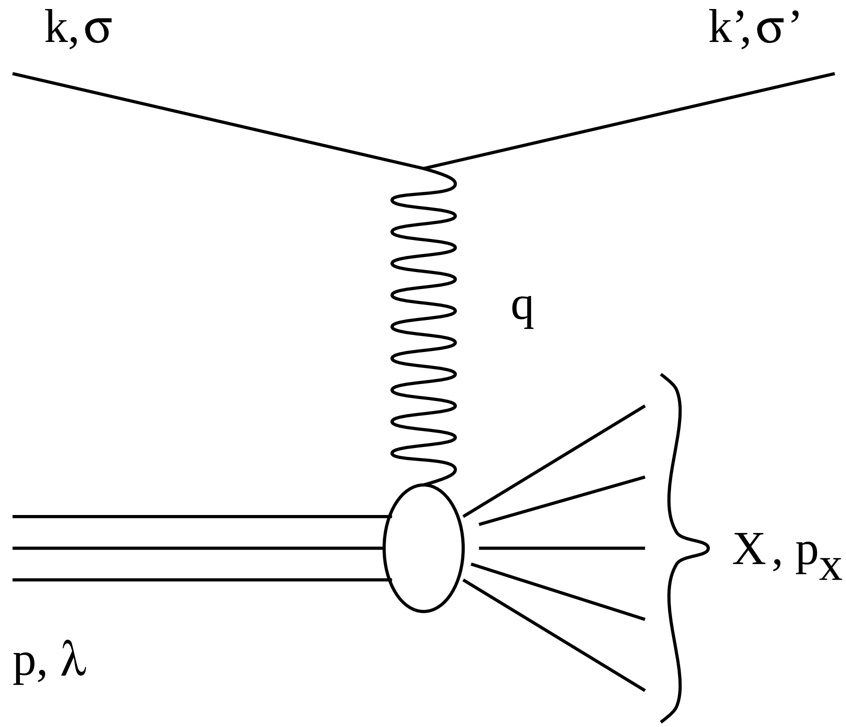

Figure 1

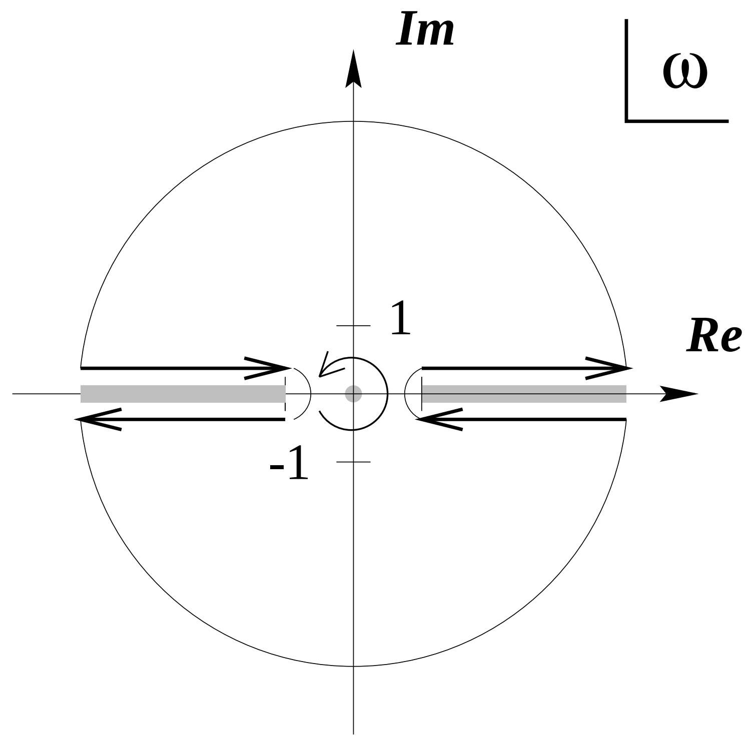

Figure 2

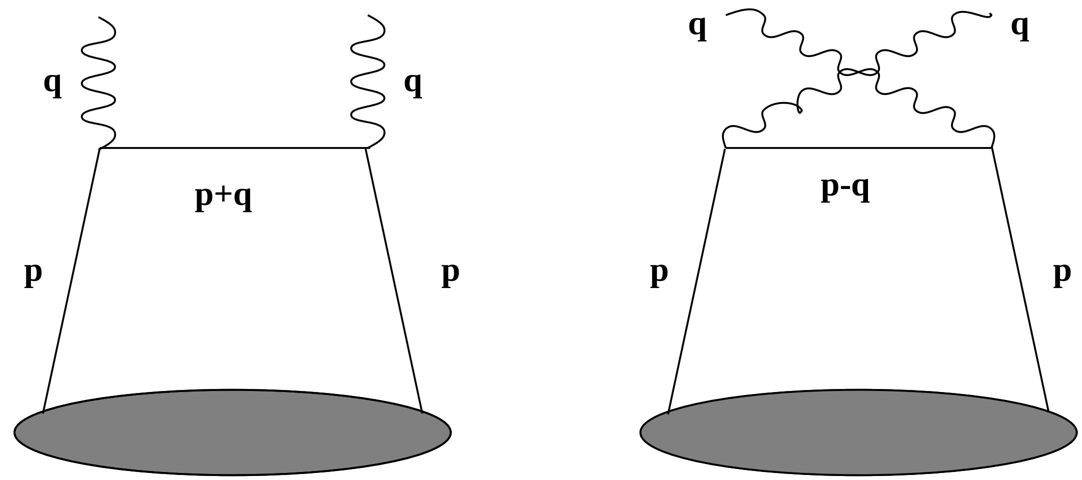

Figure 3

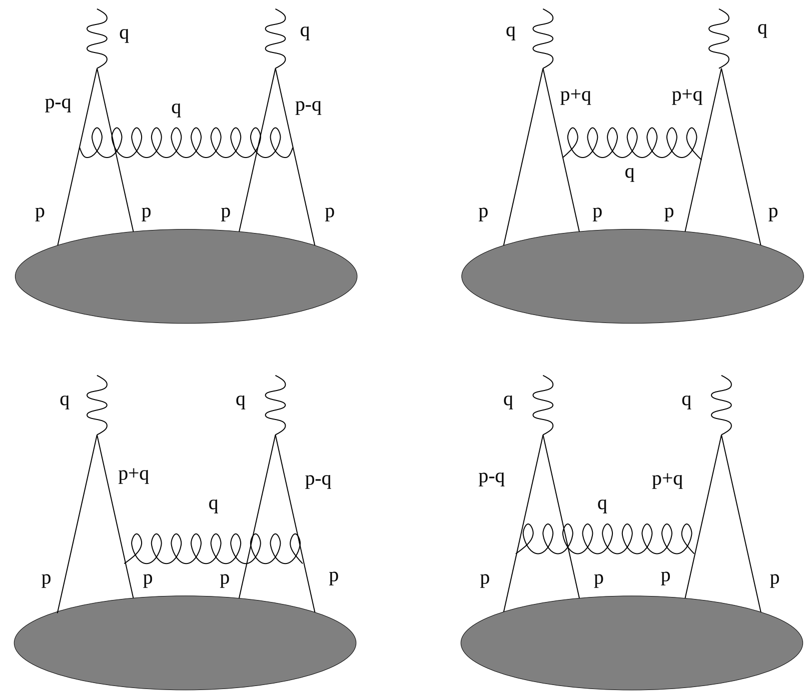

Figure 4